11. First we rearrange the equation into standard form

$$\begin{aligned}
3x^2 + 3y^2 + 3z^2 + 2y - 2z &= 9 \\
x^2 + y^2 + z^2 + \tfrac{2}{3}y - \tfrac{2}{3}z &= 3 \\
x^2 + (y^2 + \tfrac{2}{3}y) + (z^2 - \tfrac{2}{3}z) &= 3 \\
x^2 + (y + \tfrac{1}{3})^2 - \tfrac{1}{9} + (z - \tfrac{1}{3})^2 - \tfrac{1}{9} &= 3 \\
x^2 + (y + \tfrac{1}{3})^2 + (z - \tfrac{1}{3})^2 &= \frac{29}{9} \\
(x-0)^2 + (y + \tfrac{1}{3})^2 + (z - \tfrac{1}{3})^2 &= \left(\frac{\sqrt{29}}{3}\right)^2.
\end{aligned}$$

Then it is easy to see that the centre of the sphere is $(0, -\frac{1}{3}, \frac{1}{3})$ and the radius is $\frac{\sqrt{29}}{3}$.

12. (a) We have that

$$5\mathbf{a} - 3\mathbf{b} = 5(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) - 3(3\mathbf{i} + 5\mathbf{k}) = 5\mathbf{i} - 10\mathbf{j} + 15\mathbf{k} - 9\mathbf{i} - 15\mathbf{k} = -4\mathbf{i} - 10\mathbf{j}.$$

(b) We require the vector

$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{9\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}}{\sqrt{9^2 + (-2)^2 + 6^2}} = \frac{9\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}}{\sqrt{121}} = \frac{9}{11}\mathbf{i} - \frac{2}{11}\mathbf{j} + \frac{6}{11}\mathbf{k}.$$

13. (a) Yes, this makes sense. $(\mathbf{u} \times \mathbf{v})$ is a vector, so we can take the dot product with $\mathbf{w}$.

(b) No, this does not make sense. $(\mathbf{v} \cdot \mathbf{w})$ is a number. We can't take the cross product of a vector with a number.

(c) Yes, this is ok. $(\mathbf{v} \times \mathbf{w})$ is a vector. Then we have vector cross vector.

(d) No, this does not make sense. We can not take a dot product of a vector with a number.

14. (a) $\mathbf{v} \cdot \mathbf{u} = (-\mathbf{i} + \mathbf{j}) \cdot (\sqrt{2}\mathbf{i} + \sqrt{3}\mathbf{j} + 2\mathbf{k}) = -\sqrt{2} + \sqrt{3}$.

(b) $\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{-\sqrt{2}+\sqrt{3}}{6}$.

(c) $\mathrm{proj}_{\mathbf{v}}\, \mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \left(\frac{-\sqrt{2}+\sqrt{3}}{3}\right)(-\mathbf{i} + \mathbf{j}) = \left(\frac{\sqrt{2}-\sqrt{3}}{3}\right)\mathbf{i} + \left(\frac{-\sqrt{2}+\sqrt{3}}{3}\right)\mathbf{j}$.

15. We have that $\overrightarrow{AB} = \mathbf{j} - \mathbf{k}$ and $\overrightarrow{AC} = -\mathbf{i} + 2\mathbf{k}$. We calculate that

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & -1 \\ -1 & 0 & 2 \end{vmatrix} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$$

and that

$$\text{area of the triangle } ABC = \frac{1}{2}\left\|\overrightarrow{AB} \times \overrightarrow{AC}\right\| = \frac{1}{2}\|2\mathbf{i} + \mathbf{j} + \mathbf{k}\| = \frac{1}{2}\sqrt{4+1+1} = \frac{\sqrt{6}}{2}.$$