

## İSTANBUL OKAN ÜNİVERSİTESİ MÜHENDİSLİK FAKÜLTESİ MÜHENDİSLİK TEMEL BİLİMLERİ BÖLÜMÜ

2019 - 20

MATH117 Mathematics for Architects – Homework 4 Solutions

N. Course

16. (a) 
$$x = 3 + 2t, y = -2 - t, z = 1 + 3t$$

(b) Since  $\overrightarrow{AB} = \mathbf{j} + 2\mathbf{k}$  and  $\overrightarrow{AC} = -2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ , we have normal vector

$$\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 2 \\ -2 & 3 & 3 \end{vmatrix} = -3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$$

Our plane must be -3x - 4y + 2z = k for some number k. Using point A, be calculate that

k = -3x - 4y + 2z = -6 - 16 + 10 = -12.

Hence our plane is -3x - 4y + 2z = -12 or 3x + 4y - 2z = 12.

17.) We have that

$$7 = 2x - 3z = 2(-1 + 3t) - 3(5t) = -2 + 6t - 15t = -2 - 9t$$
  

$$9 = -9t$$
  

$$t = -1$$

Hence the point of intersection is  $P(x, y, z)|_{t=-1} = P(-1 + 3t, -2, 5t)|_{t=-1} = P(-4, -2, -5).$ 

18. The point P(5, 5, -3) is on the line. The line is parallel to the vector  $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$ . We calculate that

$$\overrightarrow{OP} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 5 & -3 \\ 3 & 4 & -5 \end{vmatrix} = -13\mathbf{i} + 16\mathbf{j} + 5\mathbf{k}$$

and that the distance from the point to the line is

$$d = \frac{\left\| \overrightarrow{OP} \times \mathbf{v} \right\|}{\|\mathbf{v}\|} = \frac{\sqrt{(-13)^2 + 16^2 + 5^2}}{\sqrt{3^2 + 4^2 + (-5)^2}} = \frac{\sqrt{450}}{\sqrt{50}} = \sqrt{9} = 3$$

19. The vector  $\mathbf{n} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$  is clearly normal to the plane and the plane contains the point P(2,0,0). Note that  $\overrightarrow{PS} = 2\mathbf{j} + 3\mathbf{k}$ . Hence the distance from S to the plane is

$$d = \frac{\left| \overrightarrow{PS} \cdot \mathbf{n} \right|}{\|\mathbf{n}\|} = \frac{|2+6|}{\sqrt{2^2 + 1^2 + 2^2}} = \frac{8}{3}$$

20. The point P(10, 13, 8) lies on the line and vector  $\mathbf{v} = 2\mathbf{i} - 3\mathbf{k}$  is parallel to the line. The point  $Q(\frac{1}{2}, 0, 0)$  lies on the plane and the vector  $\mathbf{n} = 6\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$  is normal to the plane.

We calculate that

$$\overrightarrow{PQ} = -9.5\mathbf{i} - 13\mathbf{j} - 8\mathbf{k},$$
  
$$\operatorname{proj}_{\mathbf{n}} \overrightarrow{PQ} = \left(\frac{\overrightarrow{PQ} \cdot \mathbf{n}}{\|\mathbf{n}\|^2}\right)\mathbf{n} = \left(\frac{-57 - 65 - 32}{36 + 25 + 16}\right)\mathbf{n} = -2\mathbf{n} = -12\mathbf{i} - 10\mathbf{j} - 8\mathbf{k}.$$

and

 $\operatorname{proj}_{\text{plane}} P = P + \operatorname{proj}_{\mathbf{n}} \overrightarrow{PQ} = (10, 13, 8) + (-12, -10, -8) = (-2, 3, 0).$ 

Please note that  $\operatorname{proj}_{\text{plane}} P$  lies on our plane because 6(-2) + 5(3) + 4(0) = -12 + 15 + 0 = 3.

We require the line passing through the point  $\operatorname{proj}_{\operatorname{plane}} P$  in the direction v. That is the line

x = -2 + 2t, y = 3, z = -3t.