2019–20 **MATH117 Mathematics for Architects – Homework 7 Solutions** N. Course

These answers were mistakenly leaked before the deadline. While it is unlikely that any students is this class saw the leaked answers, no student should be disadvantaged: I have decided that that the fairest thing to do is to give the full 12 marks to every student who submitted this piece of homework. Please check your own answers to see if they were correct.

31. (a) $\lim_{x\to\frac{2}{5}} 5x(3x-1) = 5\left(\lim_{x\to\frac{2}{5}} x\right)\left(\lim_{x\to\frac{2}{5}} 3x-1\right) = 5\left(\frac{2}{5}\right)\left(\frac{6}{5}-1\right) = \frac{2}{5}$ by the constant multiple and product rules.

    (b) $\lim_{y\to-2} \frac{y+3}{y+6} = \frac{\lim_{y\to-2}(y+3)}{\lim_{y\to-2}(y+6)} = \frac{-2+3}{-2+6} = \frac{1}{4}$ by the quotient rule.

    (c) $\lim_{v\to1} \frac{v-1}{v^2+v-2} = \lim_{v\to1} \frac{v-1}{(v-1)(v+2)} = \lim_{v\to1} \frac{1}{v+2} = \frac{1}{\lim_{v\to1}(v+2)} = \frac{1}{1+2} = \frac{1}{3}$ by the quotient rule.

32. (a). $\frac{f(x+h)-f(x)}{h} = \frac{(4-6(x+h))-(4-6x)}{h} = \frac{4-6x-6h-4+6x}{h} = \frac{-6h}{h} = -6.$

    (b). $\lim_{h\to0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to0} -6 = -6.$

    (c). $\frac{g(x+h)-g(x)}{h} = \frac{(2(x+h)^2+8)-(2x^2+8)}{h} = \frac{2(x^2+2xh+h^2)+8-2x^2-8}{h} = \frac{2x^2+4xh+2h^2-2x^2}{h} = \frac{4xh+2h^2}{h} = 4x+2h.$

    (d). $\lim_{h\to0} \frac{g(x+h)-g(x)}{h} = \lim_{h\to0} 4x+2h = 4x.$

33. (a) $\frac{dy}{dx} = \frac{d}{dx}\left(x^{-8}\right) = -8x^{-9}.$

    (b) $f'(t) = \frac{d}{dt}\left(2t^2-3t+1\right) = 4t-3.$

    (c) $\frac{d}{du}\left(5u^{0.3}-4u^{2.2}\right) = 1.5u^{-0.7}-8.8u^{1.2}.$

34. (a) $f'(x) = \frac{d}{dx}\left(\frac{2x+3}{x-2}\right) = \frac{u'v-uv'}{v^2} = \frac{(2x+3)'(x-2)-(2x+3)(x-2)'}{(x-2)^2} = \frac{2(x-2)-(2x+3)}{(x-2)^2} = \frac{2x-4-2x-3}{(x-2)^2} = \frac{-7}{(x-2)^2}.$

    (b) Since $g'(x) = \frac{d}{dx}\left((x^2+1)(2x-3)\right) = \frac{d}{dx}\left(2x^3-3x^2+2x-3\right) = 6x^2-6x+2$ we have that $g'(2) = 6(4)-6(2)+2 = 14.$

    (c) $\frac{d}{dx}\left(\frac{x^4-x^3}{3x-1}\right) = \frac{9x^4-10x^3+3x^2}{(3x-1)^2}.$

35. (a) Since $\frac{d}{dx}\left(x^2+x^{-2}\right) = 2x-2x^{-3}$ we have that $\frac{d^2}{dx^2}\left(x^2+x^{-2}\right) = \frac{d}{dx}\left(2x-2x^{-3}\right) = 2+6x^{-4}.$

    (b) Since $g(x) = \frac{x^4-x^3}{x^2-x} = \frac{x^2(x^2-x)}{x^2-x} = x^2$, we have that $g'(x) = 2x$, $g''(x) = 2$ and $g'''(x) = 0$. Therefore $g'''(3) = 0.$