



İSTANBUL OKAN ÜNİVERSİTESİ
MÜHENDİSLİK FAKÜLTESİ
MÜHENDİSLİK TEMEL BİLİMLERİ BÖLÜMÜ

30/12/2019, 2pm

MATH117 Mathematics for Architects – Final Exam

N. Course

FORENAME:

SURNAME:

STUDENT NO:

SIGNATURE:

exam duration: **90** minutes

Please answer all 5 questions.



**Do not open the exam until you are told that you may begin.
Sınavın başladığı yüksek sesle söylenene kadar sayfayı çevirmeyin.**



1. You will have **90** minutes.
 2. The points awarded for each part, of each question, are stated next to it.
 3. All of the questions are in English. You must answer in English.
 4. You must show your working for all questions.
 5. This exam contains 4 pages. Check to see if any pages are missing.
 6. If you wish to leave before the end of the exam, give your exam script to an invigilator and leave the room quietly. You may not leave in the first 20 minutes, or in the final 10 minutes, of the exam.
 7. Switch your mobile phone off and seal it in the envelope provided. Do not open your envelope until the exam is finished or you have left the room.
 8. All communication between students, either verbally or non-verbally, is strictly forbidden. Students who finish early must leave the room without communicating with other students.
 9. Calculators, mobile phones and any digital means of communication are forbidden. The sharing of pens, erasers or any other item between students is forbidden.
 10. All bags, coats, books, notes, etc. must be placed away from your desks and away from the seats next to you. You may not access these during the exam. Take out everything that you will need before the exam starts.
 11. Any student found cheating or attempting to cheat will receive a mark of zero (0), and will be investigated according to the regulations of Yükseköğretim Kurumları Öğrenci Disiplin Yönetmeliği.
1. Sınav süresi toplam **90** dakikadır.
 2. Soruların her bölümünün kaç puan olduğu yanlarında belirtilmiştir.
 3. Tüm sorular İngilizce'dir. Cevaplarınızı İngilizce veriniz.
 4. Sonuca ulaşmak için yaptığınız işlemleri ayrıntılarıyla gösteriniz.
 5. Sınav 4 sayfadan oluşmaktadır. Lütfen eksik sayfa olup olmadığını kontrol edin.
 6. Sınav süresi sona ermeden sınavınızı teslim edip çıkmak isterseniz, sınav kağıdınızı gözetmenlerden birine veriniz ve sınav salonundan sessizce çıkınız. Sınavın ilk 20 dakikası ve son 10 dakikası içinde sınav salonundan çıkmazsınız yasaktır.
 7. Cep telefonunuzu kapatınız ve size verilen zarfın içine koyunuz. Zarfı, sınav süresi bitene kadar ya da sınav salonundan çıkana kadar açmayınız.
 8. Sınav esnasında öğrenciler arasında, sözlü ya da sözsüz, her türlü iletişim kesinlikle yasaktır. Sınavını erken bitiren öğrenciler, diğer öğrencilerle hiç bir şekilde iletişim kurmadan sessizce sınıftan çıkmalıdır.
 9. Sınav esnasında hesap makinesi, cep telefonu ve dijital bilgi alışverişi yapılan her türlü malzemelerin kullanımı ile diğer silgi, kalem, vb. alışverişlerin yapılması kesinlikle yasaktır.
 10. Çanta, palto, kitap ve ders notlarınız gibi eşyalarınız sıraların üzerinden ve yanınızdaki sandalyeden kaldırılmalıdır. Sınav süresince bu tür eşyaları kullanmanız yasaktır, bu nedenle ihtiyacınız olacak herşeyi sınav başlamadan yanınıza alınız.
 11. Her türlü sınav, ve diğer çalışmada, kopya çeken veya kopya çekme girişiminde bulunan bir öğrenci, o sınav ya da çalışmadan sıfır (0) not almış sayılır, ve o öğrenci hakkında Yükseköğretim Kurumları Öğrenci Disiplin Yönetmeliği hükümleri uyarınca disiplin kovuşturması yapılır.

1	2	3	4	5	TOTAL
20	20	20	20	20	100

Question 1 (Probability) A bag contains 20 balls numbered from 1 to 20. Two balls are chosen at random from the bag and the difference between these two numbers is calculated. For example, the difference between 3 and 17 is $|3 - 17| = 14$.

(a). [8 pts] In how many different ways can we select two balls with a difference of 12?

We can choose balls that differ by 12 in the following ways: (1, 13), (2, 14), (3, 15), ..., (8, 20), (20, 8), (19, 7), (18, 6), ... and (13, 1). So the answer is that there are **16** ways for the difference to be equal to 12.

(b). [12 pts] What is the probability that the difference between the two numbers selected is **not** 12?

First we will calculate $P(A)$, the probability that the difference is 12. Then we will calculate $1 - P(A)$

There are ${}_{20}P_2 = \frac{20!}{18!} = 20 \cdot 19 = 380$ ways of choosing 2 balls from 20 balls (if the order is important). From part (a), we know that 16 of these ways give a difference of 12. Hence

$$P(A) = \frac{16}{380} = \frac{4}{95}.$$

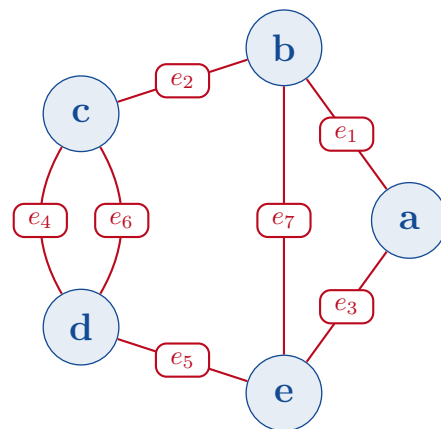
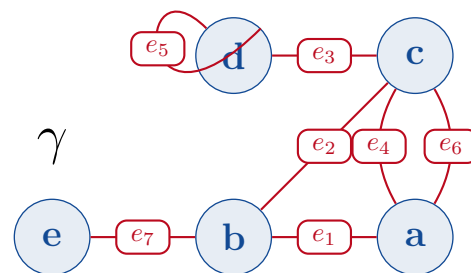
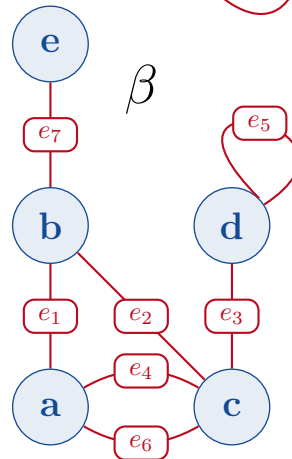
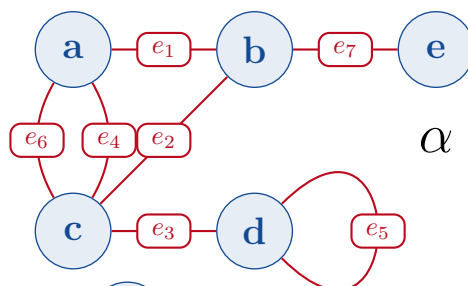
[Note: If you decide that the order of the balls is not important, then you have $\frac{8}{{}_{20}C_2}$ which is also equal to $\frac{4}{95}$.]

It follows that the probability that the balls do not differ by 12 is

$$1 - P(A) = 1 - \frac{4}{95} = \frac{91}{95}.$$

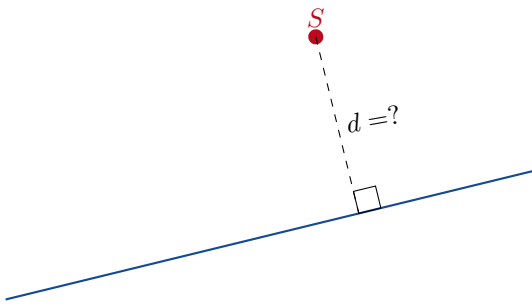
Question 2 (Graph Theory)

(a) [10 pts] Let $V = \{a, b, c, d, e\}$ and $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ where $e_1 = (a, b)$, $e_2 = (c, b)$, $e_3 = (c, d)$, $e_4 = (a, c)$, $e_5 = (d, d)$, $e_6 = (a, c)$ and $e_7 = (b, e)$. **Draw** the (undirected) graph $G = (V, E)$. [Vertices a , c and d are drawn below. You must decide where to place b and e .]



(b) [10 pts] Consider the graph above. Does this graph contain an Eulerian trail? If “yes”, **give an example** of an Eulerian trail in that graph. If “no”, **explain** how we know that it does not contain an Eulerian trail.

This graph does not contain an Eulerian trail since vertices b , c , d and e have odd degree.



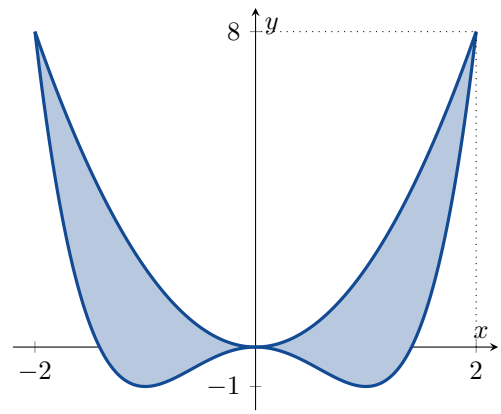
Question 3 (Lines) [20 pts] Find the distance from the point $S(2, 3, 4)$ to the line $x = 3 - t$, $y = 4 - t$, $z = 5 + 2t$.

The point $P(3, 4, 5)$ lies on the line. The line is parallel to the vector $\mathbf{v} = -\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. Note that $\overrightarrow{SP} = \mathbf{i} + \mathbf{j} + \mathbf{k}$. We calculate that

$$\overrightarrow{SP} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ -1 & -1 & 2 \end{vmatrix} = 3\mathbf{i} - 3\mathbf{j}$$

and that the distance from the point to the line is

$$\begin{aligned} d &= \frac{\|\overrightarrow{SP} \times \mathbf{v}\|}{\|\mathbf{v}\|} = \frac{\sqrt{3^2 + (-3)^2}}{\sqrt{(-1)^2 + (-1)^2 + 2^2}} \\ &= \frac{\sqrt{18}}{\sqrt{6}} = \sqrt{\frac{18}{6}} = \sqrt{3}. \end{aligned}$$



Question 4 (Calculus) [20 pts] Calculate the total area between the curve $y = 2x^2$ and the curve $y = x^4 - 2x^2$ for $-2 \leq x \leq 2$.

We calculate that

$$\begin{aligned} \text{total area} &= \int_{-2}^2 2x^2 - (x^4 - 2x^2) dx \\ &= 2 \int_0^2 4x^2 - x^4 dx \\ &= 2 \left[\frac{4}{3}x^3 - \frac{x^5}{5} \right]_0^2 \\ &= 2 \left(\left(\frac{4}{3}(8) - \frac{32}{5} \right) - (0 - 0) \right) \\ &= \frac{128}{15}. \end{aligned}$$

Question 5 (Calculus)

(a) [7 pts] Find $\int 3x(x^2 - 99)^{100} dx$.

Let $u = x^2 - 99$. Then we have that $du = \frac{du}{dx} dx = 2x dx$ and $\frac{1}{2} du = x dx$. It follows that

$$\int 3x(x^2 - 99)^{100} dx = \frac{3}{2} \int u^{100} du = \frac{3}{2} \left(\frac{u^{101}}{101} \right) + C = \frac{3}{202} (x^2 - 99)^{101} + C.$$

(b) [7 pts] Find $\frac{dy}{dx}$ if $y = \operatorname{cosec}(5x^2 - 3)$.

Let $u = 5x^2 - 3$. Then

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = \left(\frac{d}{du} \operatorname{cosec} u \right) \left(\frac{d}{dx} (5x^2 - 3) \right) = (-\operatorname{cosec} u \cot u) (10x) = -10x \operatorname{cosec}(5x^2 - 3) \cot(5x^2 - 3)$$

by the Chain Rule.

(c) [6 pts] Find $f''(0)$ if $f(x) = x \sin x$.

We calculate that

$$\begin{aligned} f'(x) &= u'v + uv' = (x)' \sin x + x(\sin x)' = \sin x + x \cos x \\ f''(x) &= (\sin x)' + (x)' \cos x + x(\cos x)' = \cos x + \cos x - x \sin x = 2 \cos x - x \sin x \end{aligned}$$

and

$$f''(0) = 2 \cos 0 - 0 \sin 0 = 2$$

by the product rule.