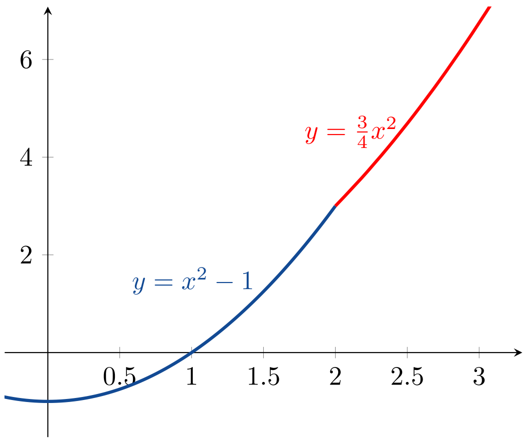
6. Clearly $|x|+1$ and $\frac{1}{2}x^2$ are continuous everywhere, because all polynomials are continuous functions and $|x|$ is a continuous function. Since $|x|+1 \neq 0$, the function $g(x) = \frac{1}{|x|+1} - \frac{x^2}{2}$ must be continuous everywhere too.

7. Because sin is continuous everywhere, it follows that

$$\begin{aligned}
\lim_{x\to\pi} \sin\Big(x - \sin\big(x - \sin(x - \sin x)\big)\Big) &= \sin\Big(\lim_{x\to\pi} x - \lim_{x\to\pi} \sin\big(x - \sin(x - \sin x)\big)\Big) \\
&= \sin\Big(\pi - \sin\Big(\lim_{x\to\pi} x - \lim_{x\to\pi} \sin(x - \sin x)\Big)\Big) \\
&= \sin\Big(\pi - \sin\Big(\pi - \sin\Big(\lim_{x\to\pi} x - \lim_{x\to\pi} \sin x\Big)\Big)\Big) \\
&= \sin\Big(\pi - \sin\Big(\pi - \sin\Big(\pi - \sin \lim_{x\to\pi} x\Big)\Big)\Big) \\
&= \sin\left(\pi - \sin\left(\pi - \sin\left(\pi - \sin\pi\right)\right)\right) \\
&= \sin\left(\pi - \sin\left(\pi - \sin\left(\pi - 0\right)\right)\right) \\
&= \sin\left(\pi - \sin\left(\pi - 0\right)\right) \\
&= \sin\left(\pi - 0\right) \\
&= 0.
\end{aligned}$$

8. Clearly f is continuous if $x \neq 2$. We want $3 = x^2 - 1\big|_{x=2} = f(2) = b(2)^2 = 4b$. So we choose $b = \frac{3}{4}$. Then f is continuous at every x.



9. (a)

$$\begin{aligned}
\lim_{x\to\infty}\left(\sqrt{x^2+x} - \sqrt{x^2-x}\right) &= \lim_{x\to\infty}\left(\sqrt{x^2+x} - \sqrt{x^2-x}\right)\frac{\sqrt{x^2+x}+\sqrt{x^2-x}}{\sqrt{x^2+x}+\sqrt{x^2-x}} \\
&= \lim_{x\to\infty}\frac{(x^2+x)-(x^2-x)}{\sqrt{x^2+x}+\sqrt{x^2-x}} = \lim_{x\to\infty}\frac{2x}{\sqrt{x^2+x}+\sqrt{x^2-x}} \\
&= \lim_{x\to\infty}\frac{2}{\sqrt{1+\frac{1}{x}}+\sqrt{1-\frac{1}{x}}} = \frac{2}{\sqrt{1+0}+\sqrt{1-0}} = 1.
\end{aligned}$$

(b)

$$\lim_{x\to-\infty}\frac{4-3x^2}{\sqrt{x^6+6x^3+9}} = \lim_{x\to-\infty}\frac{4-3x^2}{\sqrt{(x^3+3)^2}} = \lim_{x\to-\infty}\frac{4-3x^2}{x^3+3} = \lim_{x\to-\infty}\frac{\frac{4}{x^3}-\frac{3}{x}}{1+\frac{3}{x^3}} = \frac{0-0}{1+0} = 0.$$

10. (a) $\lim_{x\to-5}\dfrac{3x}{2x+10}$ does not exist. If $x > -5$ and x tends to -5, then $\frac{3x}{2x+10}$ tends to $-\infty$. But if $x < -5$ and x tends to -5, then $\frac{3x}{2x+10}$ tends to $+\infty$. Therefore the limit does not exist.

(b)

$$\lim_{p\to0}\frac{1}{p^{\frac{2}{3}}} = \lim_{p\to0}\sqrt[3]{\frac{1}{p^2}} = \sqrt[3]{\lim_{p\to0}\frac{1}{p^2}} = \infty.$$