

11. (a) Since

$$y' = (3x^7 - 7x^3 + 21x^2)' = 21x^6 - 21x^2 + 42x,$$

we have that

$$y'' = (21x^6 - 21x^2 + 42x)' = 126x^5 - 42x + 42.$$

(b) We calculate that

$$\frac{db}{dz} = \frac{d}{dx} (-2z^{-1} + 4z^{-2}) = 2z^{-2} - 8z^{-3} = \frac{2}{z^2} - \frac{8}{z^3}.$$

12. (a) We have

$$\frac{dr}{d\theta} = \frac{d}{d\theta} ((1 + \sec \theta) \sin \theta) = \frac{d}{d\theta} (\sin \theta + \tan \theta) = \cos \theta + \sec^2 \theta.$$

(b) We calculate that

$$\begin{aligned} \frac{d}{dx} (-2 \sin x) &= -2 \cos x, \\ \frac{d^2}{dx^2} (-2 \sin x) &= (-2 \cos x)' = 2 \sin x, \\ \frac{d^3}{dx^3} (-2 \sin x) &= (2 \sin x)' = 2 \cos x, \\ \frac{d^3}{dx^3} (-2 \sin x) \Big|_{x=\pi} &= 2 \cos \pi = -2. \end{aligned}$$

13. Since $\frac{dy}{dx} = \frac{d}{dx} \sin(\pi x) = \pi \cos \pi x$, the slope of the graph at $(\frac{1}{6}, \frac{1}{2})$ is $\frac{dy}{dx} \Big|_{x=\frac{1}{6}} = \pi \cos \frac{\pi}{6} = \frac{\pi\sqrt{3}}{2}$.

14. First note that

$$y = \frac{x^2 + 3}{(x-1)^3 + (x+1)^3} = \frac{x^2 + 3}{2x^3 + 6x} = \frac{1}{2x} = \frac{1}{2}x^{-1}.$$

Then it follows that

$$\frac{dy}{dx} = -\frac{1}{2}x^{-2} = \frac{-1}{2x^2}.$$

15. (a) By the chain rule, we have that

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left((5-2x)^{-3} + \frac{1}{8} \left(\frac{2}{x} + 1 \right)^4 \right) \\ &= -3(5-2x)^{-4} \frac{d}{dx} (5-2x) + \frac{4}{8} \left(\frac{2}{x} + 1 \right)^3 \frac{d}{dx} \left(\frac{2}{x} + 1 \right) \\ &= \frac{6}{(5-2x)^4} + \frac{1}{2} \left(\frac{2}{x} + 1 \right)^3 \left(\frac{-2}{x^2} \right) \\ &= \frac{6}{(5-2x)^4} - \left(\frac{1}{x^2} \right) \left(\frac{2}{x} + 1 \right)^3 \\ &= \frac{6}{(5-2x)^4} - \frac{(2+x)^3}{x^5}. \end{aligned}$$

(b) Again using the chain rule, we have that

$$\begin{aligned} p' &= \frac{d}{dt} (\sqrt{3t + t \sin 3t}) \\ &= \left(\frac{1}{2\sqrt{3t + t \sin 3t}} \right) \frac{d}{dt} (3t + t \sin 3t) \\ &= \left(\frac{1}{2\sqrt{3t + t \sin 3t}} \right) (3 + \sin 3t + 3t \cos 3t) \\ &= \left(\frac{3 + \sin 3t + 3t \cos 3t}{2\sqrt{3t + t \sin 3t}} \right). \end{aligned}$$