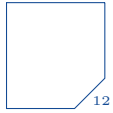


FORENAME: SURNAME: STUDENT NO: 

2018–19 Autumn

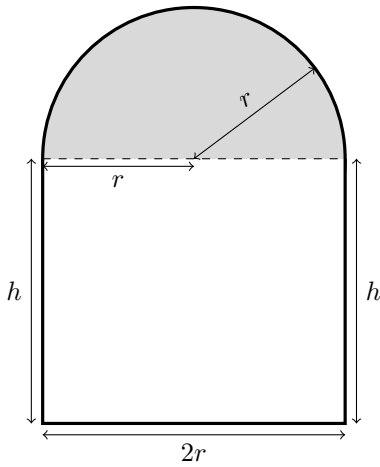
MATH115 Basic Mathematics – Homework 4

N. Course

**DEADLINE: Tuesday 30 October 2018, 3pm**

**Exercise 16 (Applied Optimisation).** You are designing a window in the form of a rectangle surmounted by a semicircle. The rectangle will be made of clear glass. The semicircle will be made of a tinted glass that transmits only half as much light, per unit area, as clear glass does. The total perimeter of the window must be 2 metres.

Find the measurements of the window ( $r$  and  $h$ ) that will admit the most light.



**Exercise 17 (Concavity and Curve Sketching).**

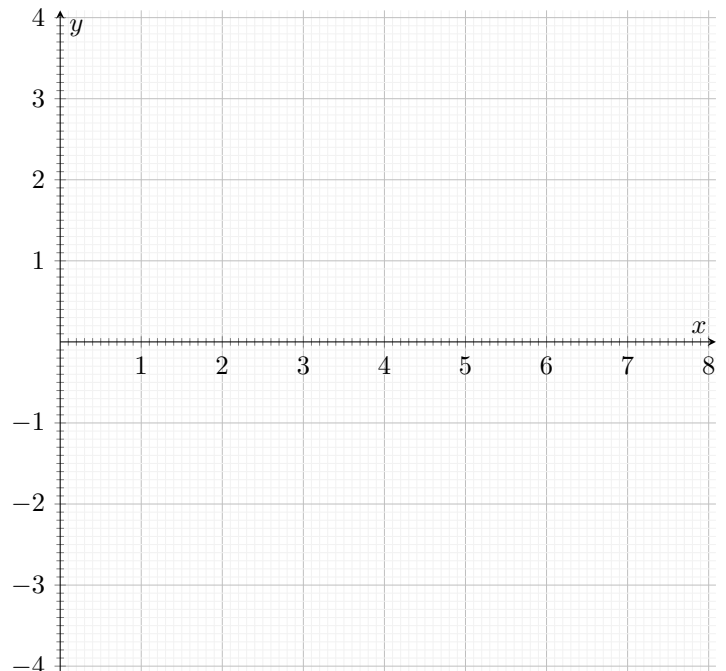
Consider the function  $f : [0, \infty) \rightarrow \mathbb{R}$ ,  $f(x) = 5x^{\frac{2}{5}} - 2x$ . Note that  $f'(x) = \frac{2}{x^{\frac{3}{5}}} - 2$  and  $f''(x) = -\frac{6}{5x^{\frac{8}{5}}}$ .

(a) [1 pts] Find all the critical points (if any) of  $y = f(x)$ .

(b) [1 pts] Find the intervals where  $f$  is increasing/decreasing.

(c) [1 pts] Find the intervals where  $f$  is concave up/down.

(d) [3 pts] Draw the graph of  $y = f(x)$  (without using a computer/a calculator/a phone/the internet/etc.).



**Exercise 18 (Antiderivatives).** Find an antiderivative for each function, then check your answer by differentiating it. The first one is done for you.

( $\omega$ )  $f(x) = 2x$ . *solution:*  $F(x) = x^2$  is an antiderivative of  $f(x) = 2x$ , because  $F'(x) = \frac{d}{dx}(x^2) = 2x = f(x)$ .

(a)  $g(x) = x^3 - \frac{1}{x^3}$ .

(b)  $h(x) = 4 \sec 3x \tan 3x$ .

(c)  $l(x) = \frac{1}{2}(e^x + e^{-x})$ .

**Exercise 19 (Right or Wrong?).** Consider  $\int (xe^x - \operatorname{cosec}^2 3x) dx = xe^x + 3 \cot 3x + C$ . Is this correct or incorrect? Why?

**Exercise 20 (Indefinite Integrals/General Antiderivatives).** Find the following indefinite integrals. The first one is done for you.

( $\omega$ ) Find  $\int 2x dx$ . *solution:*  $\int 2x dx = x^2 + C$ .

(a) Find  $\int \left(8t^3 - \frac{t^2}{2} + t\right) dt$ .

(b) Find  $\int (\sec^2 \pi\theta) d\theta$ .

(c) Find  $\int \left(x + \frac{1}{x}\right)^2 dx$ .

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I declare that this assignment is entirely my own work. I did not copy from another student and I did not allow anyone to copy from me. *Bu ödevin tamamen kendi çalışmamın ürünü olduğunu, başka bir öğrencinin ödevini kopyalamadığımı; başkasının da benim çalışmamı kopyalamasına izin vermediğimi beyan ederim.*

SIGNATURE: