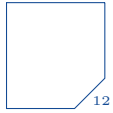2018–19 Autumn **MATH115 Basic Mathematics – Homework 4** N. Course

DEADLINE: Tuesday 30 October 2018, 3pm

Exercise 16 (Applied Optimisation). You are designing a window in the form of a rectangle surmounted by a semicircle. The rectangle will be made of clear glass. The semicircle will be made of a tinted glass that transmits only half as much light, per unit area, as clear glass does. The total perimeter of the window must be 2 metres.

Find the measurements of the window (r and h) that will admit the most light.
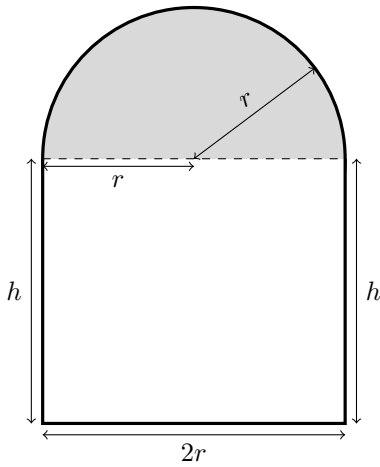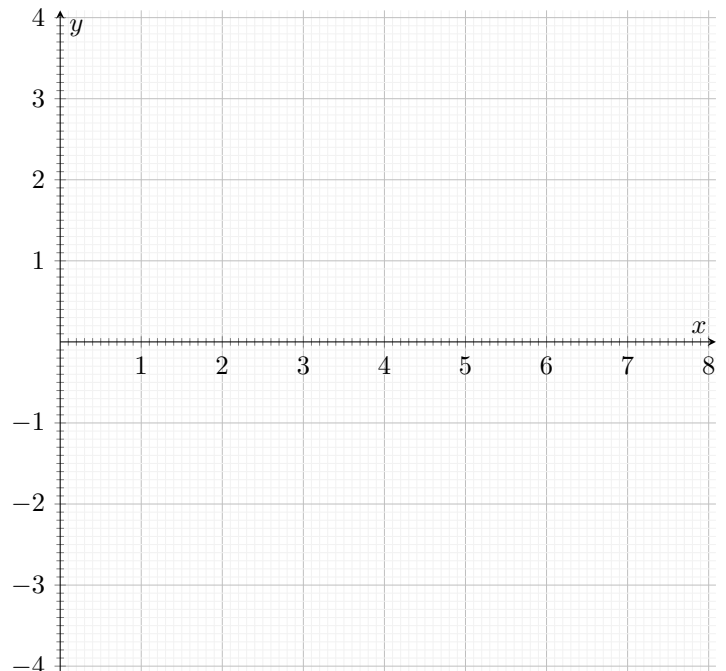

Exercise 17 (Concavity and Curve Sketching).
Consider the function $f : [0, \infty) \to \mathbb{R}$, $f(x) = 5x^{\frac{2}{5}} - 2x$.
Note that $f'(x) = \frac{2}{x^{\frac{3}{5}}} - 2$ and $f''(x) = -\frac{6}{5x^{\frac{8}{5}}}$.

(a) [1 pts] Find all the critical points (if any) of $y = f(x)$.

(b) [1 pts] Find the intervals where f is increasing/decreasing.

(c) [1 pts] Find the intervals where f is concave up/down.

(d) [3 pts] Draw the graph of $y = f(x)$ (without using a computer/a calculator/a phone/the internet/etc.).

Exercise 18 (Antiderivatives). Find an antiderivative for each function, then check your answer by differentiating it. The first one is done for you.

(ω) $f(x) = 2x$.   solution: $F(x) = x^2$ is an antiderivative of $f(x) = 2x$, because $F'(x) = \frac{d}{dx}(x^2) = 2x = f(x)$.

(a) $g(x) = x^3 - \dfrac{1}{x^3}$.

(b) $h(x) = 4\sec 3x \tan 3x$.

(c) $l(x) = \dfrac{1}{2}(e^x + e^{-x})$.

Exercise 19 (Right or Wrong?). Consider $\displaystyle\int \left(xe^x - \operatorname{cosec}^2 3x\right)\, dx = xe^x + 3\cot 3x + C$. Is this correct or incorrect? Why?

Exercise 20 (Indefinite Integrals/General Antiderivatives). Find the following indefinite integrals. The first one is done for you.

(ω) Find $\displaystyle\int 2x\, dx$.   solution: $\displaystyle\int 2x\, dx = x^2 + C$.

(a) Find $\displaystyle\int \left(8t^3 - \dfrac{t^2}{2} + t\right) dt$.

(b) Find $\displaystyle\int \left(\sec^2 \pi\theta\right)\, d\theta$.

(c) Find $\displaystyle\int \left(x + \dfrac{1}{x}\right)^2 dx$.