21. (a) We calculate that

$$\int_0^{\pi} (1+\cos x)\, dx = \Big[x+\sin x\Big]_0^{\pi} = (\pi+\sin\pi)-(0+\sin 0) = (\pi+0)-(0+0) = \pi,$$

(b) that

$$\int_{-3}^{-1} \frac{y^5-2y}{y^3}\, dy = \int_{-3}^{-1} y^2 - 2y^{-2}\, dy = \left[\frac{y^3}{3}+2y^{-1}\right]_{-3}^{-1} = \left(\frac{(-1)^3}{3}+\frac{2}{-1}\right) - \left(\frac{(-3)^3}{3}+\frac{2}{-3}\right)$$
$$= \left(-\frac{1}{3}-2\right) - \left(-9-\frac{2}{3}\right) = -\frac{1}{3}-\frac{6}{3}+\frac{27}{3}+\frac{2}{3} = \frac{22}{3},$$

(c) and that

$$\int_1^2 \left(t^2+\sqrt{t}\right)\, dt = \left[\frac{t^3}{3}+\frac{2t^{\frac{3}{2}}}{3}\right]_1^2 = \left(\frac{8}{3}+\frac{4\sqrt{2}}{3}\right) - \left(\frac{1}{3}+\frac{2}{3}\right) = \frac{5+4\sqrt{2}}{3}.$$

22. Let $u = \tan x$. Then by the Fundamental Theorem of Calculus and the Chain Rule, we have that

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{d}{du}\left(\int_u^0 \frac{1}{1+t^2}\, dt\right)\frac{d}{dx}(\tan x) = \frac{d}{du}\left(-\int_0^u \frac{1}{1+t^2}\, dt\right)\frac{d}{dx}(\tan x)$$
$$= \left(-\frac{1}{1+u^2}\right)(\sec^2 x) = -\frac{\sec^2 x}{1+\tan^2 x} = -\frac{\sec^2 x}{\sec^2 x} = -1.$$

23. (a) Let $u = 5x+8$. Then $\frac{du}{dx} = 5$ and hence $du = 5\, dx$ and $dx = \frac{1}{5}\, du$. Therefore

$$\int \frac{1}{\sqrt{5x+8}}\, dx = \frac{1}{5}\int u^{-\frac{1}{2}}\, du = \frac{1}{5}\left(2u^{\frac{1}{2}}\right)+C = \frac{2}{5}\sqrt{u}+C = \frac{2}{5}\sqrt{5x+8}+C.$$

(b) Now let $u = \tan\frac{x}{2}$. Then $\frac{du}{dx} = \frac{1}{2}\sec^2\frac{x}{2}$ and $2\, du = \sec^2\frac{x}{2}\, dx$. Hence

$$\int \tan^7\frac{x}{2}\sec^2\frac{x}{2}\, dx = \int 2u^7\, du = \frac{1}{4}u^8 + C = \frac{1}{4}\tan^8\frac{x}{2}+C.$$

24. (a) Let $u = \cos x$. Then $\frac{du}{dx} = -\sin x$ and $-du = \sin x\, dx$. Moreover $x = 2\pi \implies u = \cos 2\pi = 1$ and $x = 3\pi \implies u = \cos 3\pi = -1$. Therefore

$$\int_{2\pi}^{3\pi} 3\cos^2 x \sin x\, dx = \int_1^{-1} -3u^2\, du = \int_{-1}^{1} 3u^2\, du = \Big[u^3\Big]_{-1}^{1} = 1^3-(-1)^3 = 2.$$

(b) Let $u = 1+\sqrt{y}$. Then $\frac{du}{dy} = \frac{1}{2\sqrt{y}}\, dy$ and $du = \frac{1}{2\sqrt{y}}\, dx$. Moreover $y = 1 \implies u = 1+\sqrt{1} = 2$ and $y = 4 \implies u = 1+\sqrt{4} = 3$. Hence

$$\int_1^4 \frac{1}{2\sqrt{y}(1+\sqrt{y})^2}\, dy = \int_2^3 \frac{1}{u^2}\, du = \int_2^3 u^{-2}\, du = \Big[-u^{-1}\Big]_2^3 = \left(-\frac{1}{3}\right) - \left(-\frac{1}{2}\right) = \frac{1}{6}.$$

25. We must integrate $(x^2) - (-2x^4) = x^2+2x^4$ between $x = 1$ and $x = 1$. Thus

$$\textbf{area} = \int_{-1}^{1} x^2+2x^4\, dx = 2\int_0^1 x^2+2x^4\, dx = 2\left[\frac{x^3}{3}+\frac{2x^5}{5}\right]_0^1 = 2\left(\left(\frac{1}{3}+\frac{2}{5}\right)-(0+0)\right) = \frac{22}{15}.$$