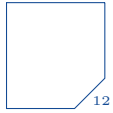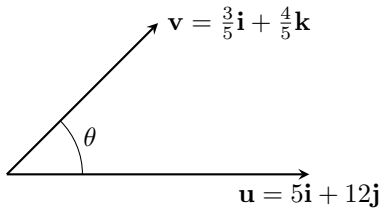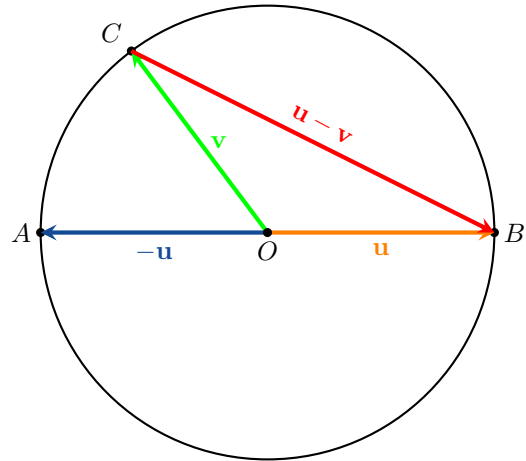**DEADLINE: Tuesday 11 December 2018, 3pm**

Exercise 31 (The Dot Product). Let $\mathbf{u} = 5\mathbf{i} + 12\mathbf{j}$ and $\mathbf{v} = \frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{k}$. Let θ denote the angle between $\mathbf{u}$ and $\mathbf{v}$.

$\mathbf{v} = \frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{k}$

θ

$\mathbf{u} = 5\mathbf{i} + 12\mathbf{j}$

(a) Find $\mathbf{v} \cdot \mathbf{u}$.

(b) Find $\cos\theta$.

(c) Find $\operatorname{proj}_{\mathbf{v}} \mathbf{u}$.

Exercise 32 (Properties of the Dot Product).
Suppose that points A and B lie on opposite sides of a circle with centre O. Let C be another point on the circle.

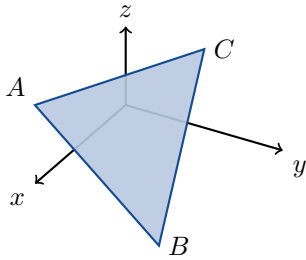Let $\mathbf{u} = \overrightarrow{OB}$ and $\mathbf{v} = \overrightarrow{OC}$.

Note that $\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC} = \mathbf{u} - \mathbf{v}$. Moreover, $\|u\| = \|v\| =$ the radius of the circle.

(a) Find a formula for $\overrightarrow{CA}$ in terms of $\mathbf{u}$ and $\mathbf{v}$.

(b) Using your answer to part (a), and the information above, show that $\overrightarrow{CA} \cdot \overrightarrow{CB} = 0$.

(c) Find the angle between $\overrightarrow{CA}$ and $\overrightarrow{CB}$.

Exercise 33 (The Cross Product).   Find the area of the triangle with vertices at $A(2,0,1)$, $B(1,1,-1)$ and $C(0,1,1)$.

Exercise 34 (True or False).   Let $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ be vectors. Which of the following are always true, and which are not always true? Give reasons for your answers. The first one is done for you.

(ω) $\|\mathbf{u}\| = \sqrt{\mathbf{u} \cdot \mathbf{u}}$ *Solution:* This is always true because $\mathbf{u} \cdot \mathbf{u} = \|\mathbf{u}\|^2$ is a rule.

(a) $(\mathbf{u} \times \mathbf{u}) \cdot \mathbf{u} = 0$

(b) $\mathbf{u} \cdot \mathbf{u} = \|\mathbf{u}\|$

(c) $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = \mathbf{v} \cdot (\mathbf{u} \times \mathbf{v})$

(d) $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$

Exercise 35 (The Cross Product).
   (a) Calculate $\big((\mathbf{i} \times \mathbf{j}) \times \mathbf{j}\big) - \big(\mathbf{i} \times (\mathbf{j} \times \mathbf{j})\big)$.

   (b) Calculate $(\tfrac{3}{2}\mathbf{i} - \tfrac{1}{2}\mathbf{j} + \mathbf{k}) \times (\mathbf{i} + \mathbf{j} + 2\mathbf{k})$.