OKAN ÜNİVERSİTESİ
MÜHENDİSLİK FAKÜLTESİ
MÜHENDİSLİK TEMEL BİLİMLERİ BÖLÜMÜ

20–12–2017, 9:30-11:30 | **MATH115 Basic Mathematics – Final Exam** | N. Course

FORENAME:

SURNAME:

STUDENT NO:

SIGNATURE:

Time Allowed: **120** min.

Answer **4** questions.

Do not open the exam until you are told that you may begin.
Sınavın başladığı yüksek sesle söylenene kadar sayfayı çevirmeyin.

1. You will have **120** minutes to answer **4** questions from a choice of 5. If you choose to answer more than 4 questions, then only your best 4 answers will be counted.
2. The points awarded for each part, of each question, are stated next to it.
3. All of the questions are in English. You must answer in English.
4. You must show your working for all questions.
5. This exam contains 8 pages. Check to see if any pages are missing.
6. If you wish to leave before the end of the exam, give your exam script to an invigilator and leave the room quietly. You may not leave in the first 20 minutes, or in the final 10 minutes, of the exam.
7. Switch your mobile phone off and seal it in the envelope provided. Do not open your envelope until the exam is finished or you have left the room.
8. Calculators, mobile phones and any digital means of communication are forbidden. The sharing of pens, erasers or any other item between students is forbidden.
9. All bags, coats, books, notes, etc. must be placed away from your desks and away from the seats next to you. You may not access these during the exam. Take out everything that you will need before the exam starts.
10. Any student found cheating or attempting to cheat will receive a mark of zero (0), and will be investigated according to the regulations of Yükseköğretim Kurumları Öğrenci Disiplin Yönetmeliği.

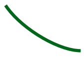
1. Sınav süresi toplam **120** dakikadır. Sınavda 5 soru sorulmuştur. Bu sorulardan **4** tanesini seçerek cevaplayınız. 4'den fazla soruyu cevaplarsanız, en yüksek puanı aldığınız 4 sorunun cevapları geçerli olacaktır.
2. Soruların her bölümünün kaç puan olduğu yanlarında belirtilmiştir.
3. Tüm sorular İngilizce'dir. Cevaplarınızı İngilizce veriniz.
4. Sonuca ulaşmak için yaptığınız işlemleri ayrıntılarıyla gösteriniz.
5. Sınav 8 sayfadan oluşmaktadır. Lütfen eksik sayfa olup olmadığını kontrol edin.
6. Sınav süresi sona ermeden sınavınızı teslim edip çıkmak isterseniz, sınav kağıdınızı gözetmenlerden birine veriniz ve sınav salonundan sessizce çıkınız. Sınavın ilk 20 dakikası ve son 10 dakikası içinde sınav salonundan çıkmanız yasaktır.
7. Cep telefonunuzu kapatınız ve size verilen zarfın içine koyunuz. Zarfı, sınav süresi bitene kadar ya da sınav salonundan çıkana kadar açmayınız.
8. Sınav esnasında hesap makinesi, cep telefonu ve dijital bilgi alışverişi yapılan her türlü malzemelerin kullanımı ile diğer silgi, kalem, vb. alışverişlerin yapılması kesinlikle yasaktır.
9. Çanta, palto, kitap ve ders notlarınız gibi eşyalarınız sıraların üzerinden ve yanınızdaki sandalyeden kaldırılmalıdır. Sınav süresince bu tür eşyaları kullanmanız yasaktır, bu nedenle ihtiyacınız olacak herşeyi sınav başlamadan yanınıza alınız.
10. Her türlü sınav, ve diğer çalışmada, kopya çeken veya kopya çekme girişiminde bulunan bir öğrenci, o sınav ya da çalışmadan sıfır (0) not almış sayılır, ve o öğrenci hakkında Yükseköğretim Kurumları Öğrenci Disiplin Yönetmeliği hükümleri uyarınca disiplin kovuşturması yapılır.

1	2	3	4	5	TOTAL
/25	/25	/25	/25	/25	/100

Question 1 (Concavity and Curve Sketching) Consider the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^3 - 3x + 3$. The first two derivatives of f are $f'(x) = 3x^2 - 3$ and $f''(x) = 6x$.

Solving $0 = f(x) = x^3 - 3x + 3$ gives $x \approx -2.1038$.
Solving $0 = f'(x) = 3x^2 - 3$ gives $x = -1$ and $x = 1$.
Solving $0 = f''(x) = 6x$ gives $x = 0$.

We have that

$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
$f' > 0$	$f' < 0$	$f' < 0$	$f' > 0$
$f'' < 0$	$f'' < 0$	$f'' > 0$	$f'' > 0$
f is increasing and concave down	f is decreasing and concave down	f is decreasing and concave up	f is increasing and concave up
			

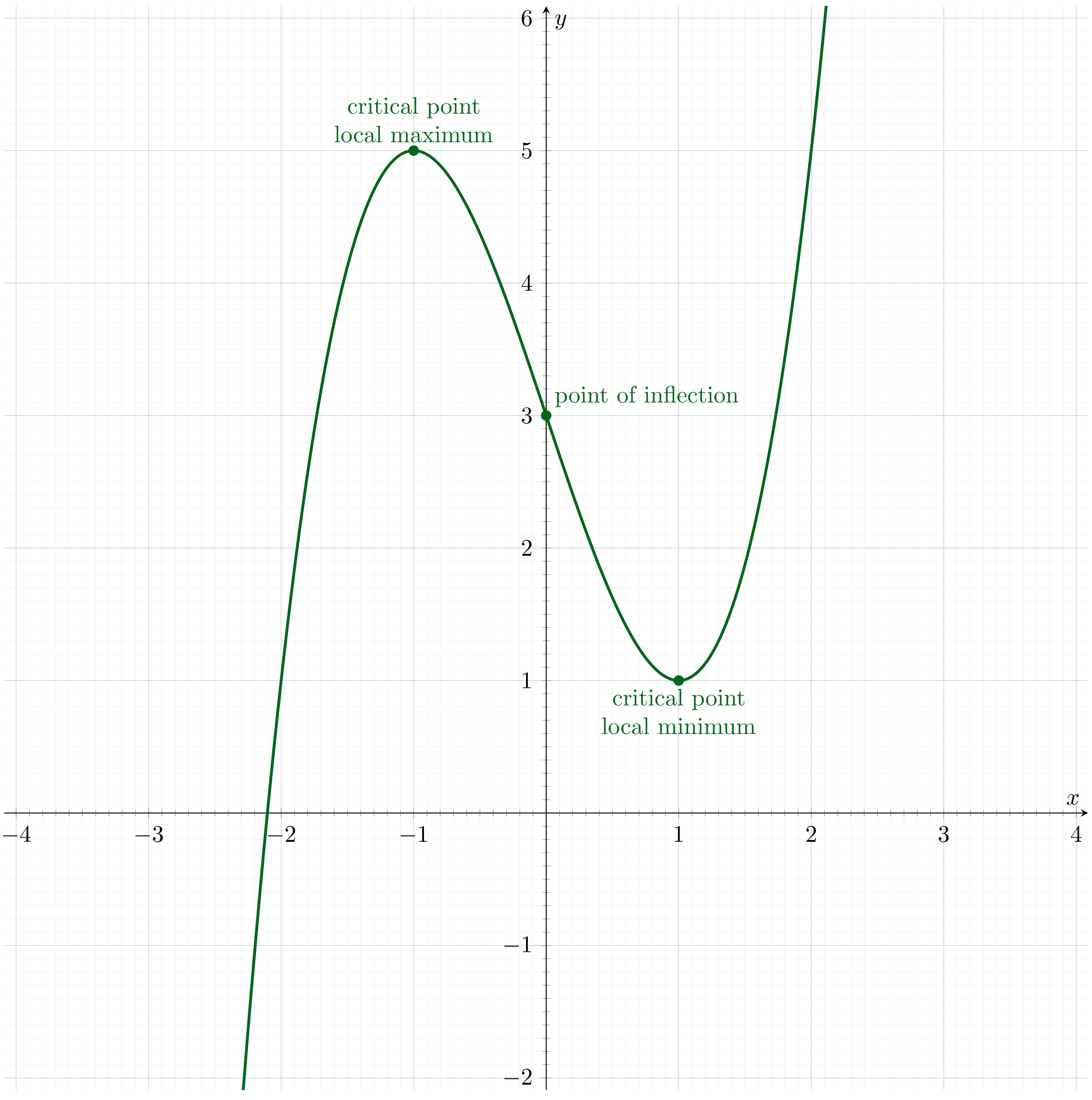
x	$f(x)$
-2	1
-1	5
0	3
1	1
2	5

(a) [16 pts] Use the information above to draw the graph of $y = f(x)$ on the axes below.
[Note that 16 points $\approx$ 16 minutes of your exam. Please take your time and draw a very neat graph. *Lütfen dikkat edin; 16 puan yaklaşık 16 dakika demektir, acele etmeden muntazam bir grafik çiziniz.*]

(b) [3 pts] Label all of the critical points on your graph.

(c) [3 pts] Label all of the points of inflection on your graph.

(d) [3 pts] Label all of the local maxima and local minima on your graph.

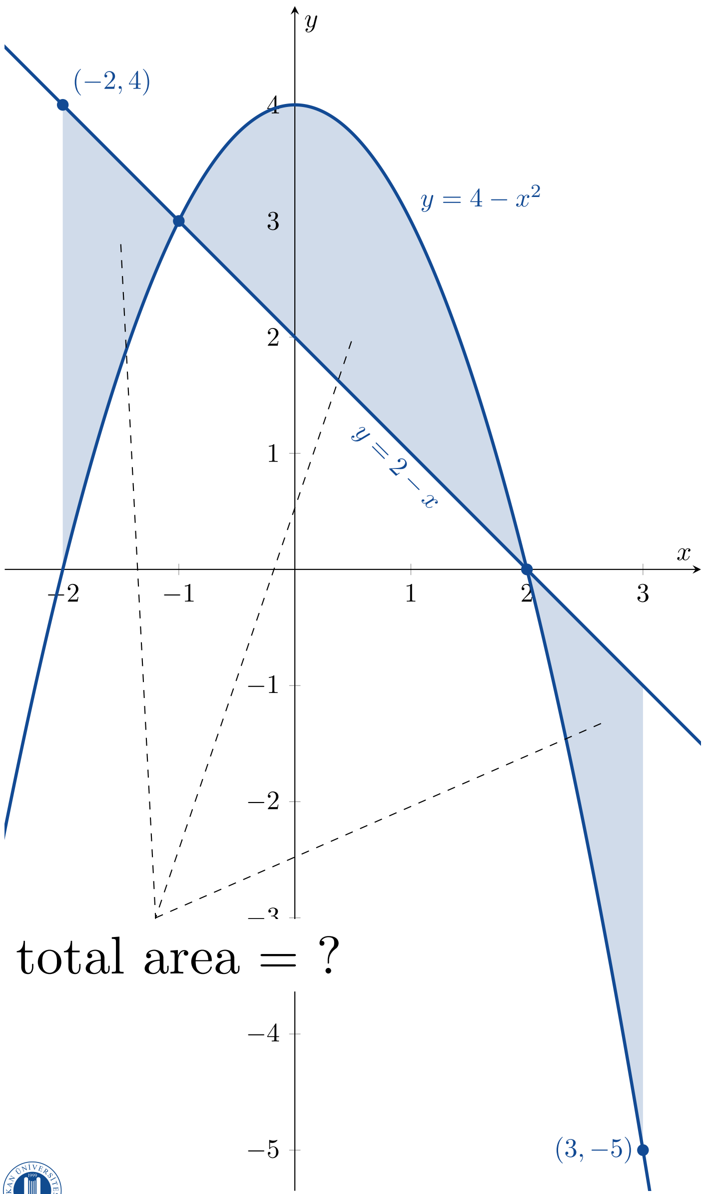


Question 2 (Area Between Curves)

(a) [1 pt] Please write your student number at the top-right of this page.

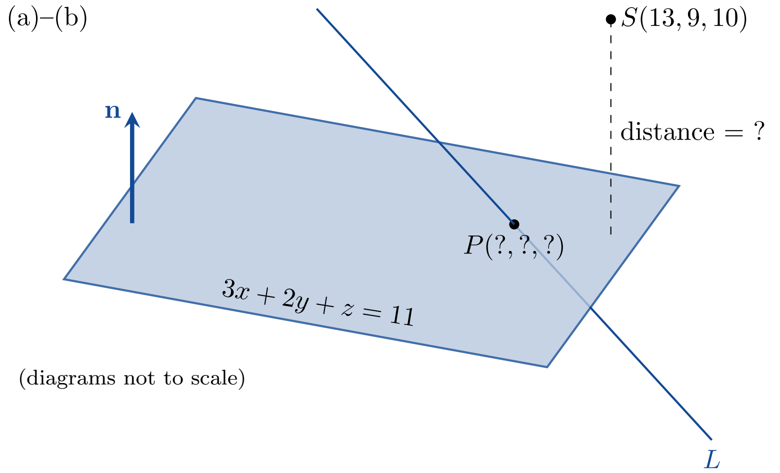
(b) [24 pts] Calculate the total area between the curves $y = 2 - x$ and $y = 4 - x^2$ for $-2 \le x \le 3$.

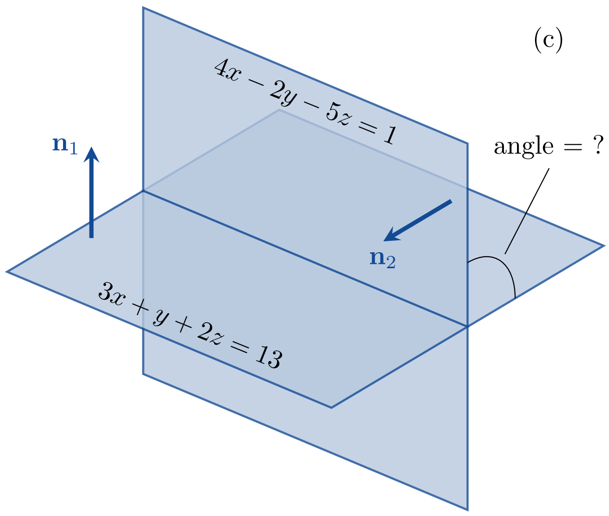
$$\begin{aligned}
\text{total area} &= \text{piece 1} + \text{piece 2} + \text{piece 3} \\
&= \int_{-2}^{-1} (2-x) - (4-x^2)\; dx + \int_{-1}^{2} (4-x^2) - (2-x)\; dx + \int_{2}^{3} (2-x) - (4-x^2)\; dx \\
&= \int_{-2}^{-1} -2 - x + x^2\; dx + \int_{-1}^{2} 2 + x - x^2\; dx + \int_{2}^{3} -2 - x + x^2\; dx \\
&= \left[-2x - \frac{1}{2}x^2 + \frac{1}{3}x^3\right]_{-2}^{-1} + \left[2x + \frac{1}{2}x^2 - \frac{1}{3}x^3\right]_{-1}^{2} + \left[-2x - \frac{1}{2}x^2 + \frac{1}{3}x^3\right]_{2}^{3} \\
&= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(4 - 2 - \frac{8}{3}\right) + \left(4 + 2 - \frac{8}{3}\right) - \left(-2 + \frac{1}{2} + \frac{1}{3}\right) + \left(-6 - \frac{9}{2} + 9\right) - \left(-4 - 2 + \frac{8}{3}\right) \\
&= 2 - \frac{3}{6} - \frac{2}{6} - 4 + 2 + \frac{16}{6} + 4 + 2 - \frac{16}{6} + 2 - \frac{3}{6} - \frac{2}{6} - 6 - \frac{27}{6} + 9 + 4 + 2 - \frac{16}{6} \\
&= 17 - \frac{53}{6} \\
&= \frac{49}{6}.
\end{aligned}$$



total area = ☐

Question 3 (Planes)





(a) [5 pts] Find the point where the line $x = 2 - t$, $y = 6 - 2t$, $z = 5 + 3t$ intersects the plane $3x + 2y + z = 11$.

We calculate that

$$\begin{aligned} 11 &= 3x + 2y + z = 3(2 - t) + 2(6 - 2t) + (5 + 3t) = 6 - 3t + 12 - 4t + 5 + 3t = 23 - 4t \\ -12 &= -4t \\ 3 &= t \end{aligned}$$

and that

$$x = 2 - t = 2 - 3 = -1, \qquad y = 6 - 2t = 6 - 6 = 0, \qquad z = 5 + 3t = 5 + 9 = 14.$$

The line intersects the plane at the point $P(-1, 0, 14)$.

point of intersection = $P\Big(\square , \square , \square \Big)$

(b) [15 pts] Find the distance from the point $S(13, 9, 10)$ to the plane $3x + 2y + z = 11$.

Clearly $\mathbf{n} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k} = (3, 2, 1)$ is normal to the plane. The point $P(-1, 0, 14)$ lines on the plane and $\overrightarrow{PS} = S - P = (13, 9, 10) - (-1, 0, 14) = (14, 9, -4)$. The distance between the point S and the plane is

$$d = \frac{\left|\overrightarrow{PS} \cdot \mathbf{n}\right|}{\|\mathbf{n}\|} = \frac{|42 + 18 - 4|}{\sqrt{9 + 4 + 1}} = \frac{56}{\sqrt{14}} = 4\sqrt{14}.$$

distance = $\square$

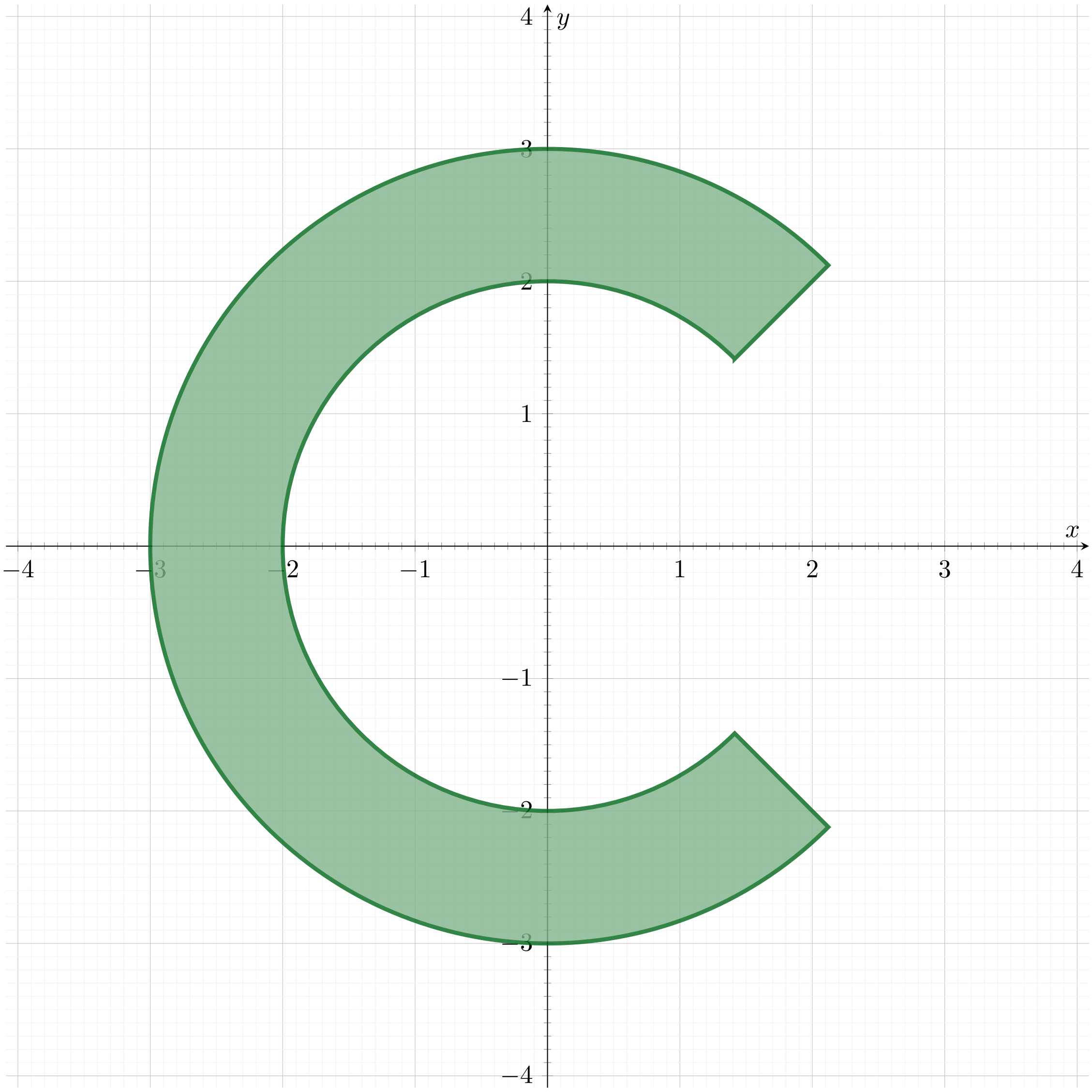
(c) [5 pts] Find the angle between the planes $3x + y + 2z = 13$ and $4x - 2y - 5z = 1$.

The vectors $\mathbf{n}_1 = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{n}_2 = 4\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}$ are normal to the first and second planes respectively. Since $\mathbf{n}_1 \cdot \mathbf{n}_2 = 12 - 2 - 10 = 0$, the normal vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ are orthogonal. Hence the angle between the planes is $90°$.

angle = ☐

Question 4 (Polar Coordinates / Spheres / Spherical Polar Coordinates)

(a) [9 pts] Draw the set of points whose polar coordinates satisfy $2 \leq r \leq 3$ and $45° \leq \theta \leq 315°$.



(question 4 continued)

(b) [8 pts] Find the centre and the radius of the sphere

$$x^2 + y^2 + z^2 - 8x - 6y + 10z + 34 = 0.$$

First we must write our equation in the standard form:

$$\begin{aligned} x^2 + y^2 + z^2 - 8x - 6y + 10z + 34 &= 0 \\ (x^2 - 8x) + (y^2 - 6y) + (z^2 + 10z) + 34 &= 0 \\ (x^2 - 8x + 16) - 16 + (y^2 - 6y + 9) - 9 + (z^2 + 10z + 25) - 25 + 34 &= 0 \\ (x-4)^2 - 16 + (y-3)^2 - 9 + (z+5)^2 - 25 + 34 &= 0 \\ (x-4)^2 + (y-3)^2 + (z+5)^2 &= 16 \\ (x-4)^2 + (y-3)^2 + (z+5)^2 &= 4^2 \end{aligned}$$

Then we can see that the centre of the sphere is $P_0(4, 3, -5)$ and the radius of the sphere is 4.

centre $= P_0\big(\square, \square, \square\big)$

radius $= \square$

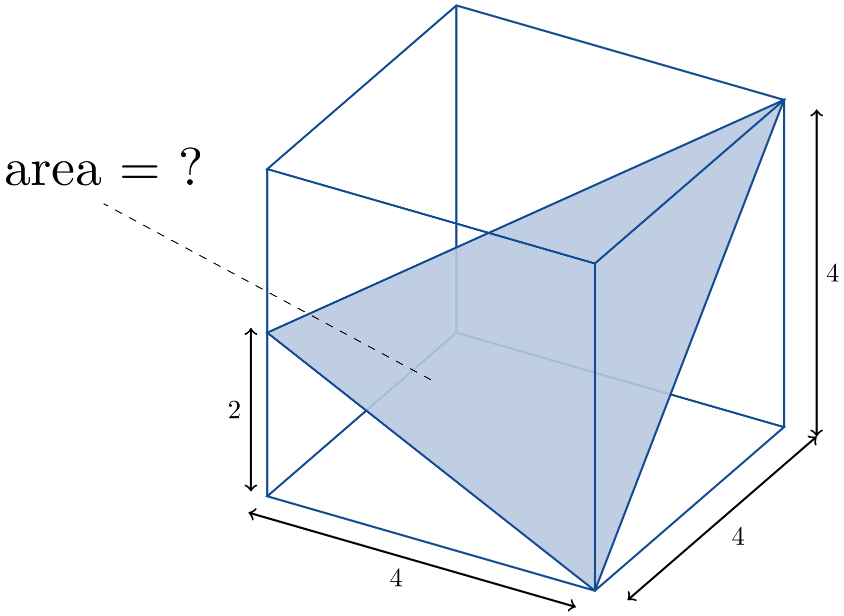
(c) [8 pts] Convert the spherical polar coordinates $(\rho, \theta, \phi) = (2, 30°, 60°)$ into Cartesian coordinates.

We calculate that

$$\begin{aligned} x &= \rho \sin\phi \cos\theta = 2 \sin 60° \cos 30° = 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{3}{2}, \\ y &= \rho \sin\phi \sin\theta = 2 \sin 60° \sin 30° = 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}, \\ z &= \rho \cos\phi = 2 \cos 60° = 2\left(\frac{1}{2}\right) = 1. \end{aligned}$$

Hence the point $(\rho, \theta, \phi) = (2, 30°, 60°)$ has Cartesian coordinates $(x, y, z) = \left(\frac{3}{2}, \frac{\sqrt{3}}{2}, 1\right)$.

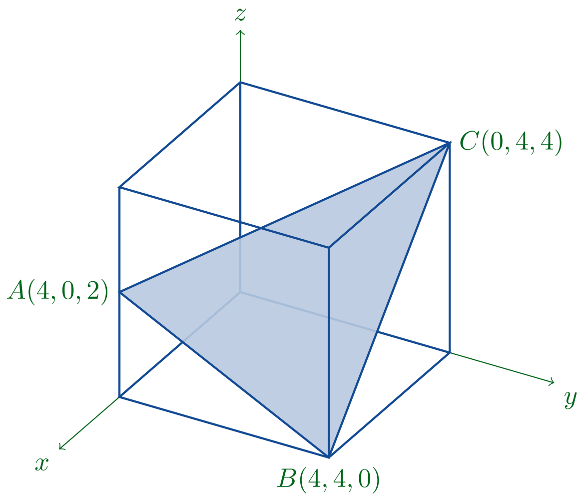
$(x, y, z) = \big(\square, \square, \square\big)$



Question 5 (The Cross Product) A triangle is inscribed inside a cube of side 4, as shown above.

[25 pts] **Use the cross product** to calculate the area of the triangle.

First we must decide where to put the origin. If we put the bottom back corner of the cube at the origin, then the three corners of the triangle are at $A(4,0,2)$, $B(4,4,0)$ and $C(0,4,4)$.



Let

$$\mathbf{u} = \overrightarrow{AB} = B - A = (4,4,0) - (4,0,2) = (0,4,-2) = 4\mathbf{j} - 2\mathbf{k}$$

and

$$\mathbf{v} = \overrightarrow{AC} = C - A = (0,4,4) - (4,0,2) = (-4,4,2) = -4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}.$$

We have that

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & -2 \\ -4 & 4 & 2 \end{vmatrix} = 16\mathbf{i} + 8\mathbf{j} + 16\mathbf{k} = 8(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$$

and

$$\|\mathbf{u} \times \mathbf{v}\| = \|8(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})\| = 8\,\|2\mathbf{i} + \mathbf{j} + 2\mathbf{k}\| = 8\sqrt{4+1+4} = 8 \times 3 = 24.$$

Therefore the area of the triangle is

$$\text{area} = \frac{1}{2}\,\|\mathbf{u} \times \mathbf{v}\| = \frac{1}{2} \times 24 = 12.$$

area of triangle = ☐

$$\cos\theta = \sin\left(\tfrac{\pi}{2} - \theta\right)$$
$$\cos^2\theta + \sin^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos^2\theta = \tfrac{1}{2}(1+\cos 2\theta)$$
$$\sin^2\theta = \tfrac{1}{2}(1-\cos 2\theta)$$

$$x = r\cos\theta \qquad y = r\sin\theta \qquad x^2 + y^2 = r^2$$

$$x = \rho\sin\phi\cos\theta \qquad y = \rho\sin\phi\sin\theta \qquad z = \rho\cos\phi \qquad \rho = \sqrt{x^2+y^2+z^2}$$

$$\cos 0 = \cos 0^\circ = 1 \qquad \sin 0 = \sin 0^\circ = 0$$
$$\cos\tfrac{\pi}{6} = \cos 30^\circ = \tfrac{\sqrt{3}}{2} \qquad \sin\tfrac{\pi}{6} = \sin 30^\circ = \tfrac{1}{2}$$
$$\cos\tfrac{\pi}{4} = \cos 45^\circ = \tfrac{1}{\sqrt{2}} \qquad \sin\tfrac{\pi}{4} = \sin 45^\circ = \tfrac{1}{\sqrt{2}}$$
$$\cos\tfrac{\pi}{3} = \cos 60^\circ = \tfrac{1}{2} \qquad \sin\tfrac{\pi}{3} = \sin 60^\circ = \tfrac{\sqrt{3}}{2}$$
$$\cos\tfrac{\pi}{2} = \cos 90^\circ = 0 \qquad \sin\tfrac{\pi}{2} = \sin 90^\circ = 1$$

$$(uv)' = uv' + u'v$$
$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$
$$(f\circ g)'(x) = f'(g(x))\,g'(x)$$

$$\tfrac{d}{dx}x^n = nx^{n-1}$$
$$\tfrac{d}{dx}\sin x = \cos x$$
$$\tfrac{d}{dx}\cos x = -\sin x$$
$$\tan x = \tfrac{\sin x}{\cos x} \qquad \tfrac{d}{dx}\tan x = \sec^2 x$$
$$\sec x = \tfrac{1}{\cos x} \qquad \tfrac{d}{dx}\sec x = \sec x\tan x$$
$$\cot x = \tfrac{\cos x}{\sin x} \qquad \tfrac{d}{dx}\cot x = -\operatorname{cosec}^2 x$$
$$\operatorname{cosec} x = \tfrac{1}{\sin x} \qquad \tfrac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x\cot x$$
$$\tfrac{d}{dx}e^x = e^x$$
$$\tfrac{d}{dx}\ln|x| = \tfrac{1}{x}$$

$$\operatorname{av}(f) = \frac{1}{b-a}\int_a^b f(x)\,dx$$
$$V = \int_a^b A(x)\,dx \qquad V = \int_a^b \pi\big(R(x)\big)^2\,dx$$

$$c = \sqrt{a^2 - b^2} \qquad \text{or} \qquad c = \sqrt{a^2 + b^2}$$

$$\operatorname{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} \qquad \theta = \cos^{-1}\left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}\right)$$
$$\mathbf{u}\times\mathbf{v} = (u_2v_3 - u_3v_2)\mathbf{i} - (u_1v_3 - u_3v_1)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k}$$
$$d = \frac{\left\|\overrightarrow{PS}\times\mathbf{v}\right\|}{\|\mathbf{v}\|} \qquad d = \frac{\left|\overrightarrow{PS}\cdot\mathbf{n}\right|}{\|\mathbf{n}\|}$$
$$d = \frac{\left\|\overrightarrow{P_1P_2}\times\mathbf{v}_1\right\|}{\|\mathbf{v}_1\|} \qquad d = \frac{\left|\overrightarrow{P_1P_2}\cdot(\mathbf{v}_1\times\mathbf{v}_2)\right|}{\|\mathbf{v}_1\times\mathbf{v}_2\|}$$