İSTANBUL OKAN ÜNİVERSİTESİ
MÜHENDİSLİK FAKÜLTESİ
MÜHENDİSLİK TEMEL BİLİMLERİ BÖLÜMÜ

2019.01.02 | **MATH115 Basic Mathematics – Final Exam** | N. Course

FORENAME:

SURNAME:

STUDENT NO:

SIGNATURE:

exam duration: **120** minutes

Please answer all 6 questions.

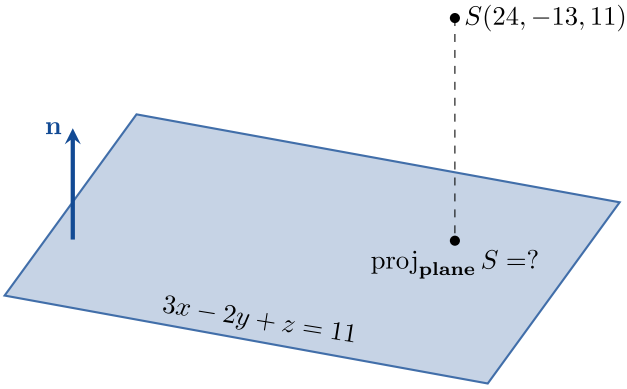
Do not open the exam until you are told that you may begin.
Sınavın başladığı yüksek sesle söylenene kadar sayfayı çevirmeyin.

1. You will have **120** minutes.
2. The points awarded for each part, of each question, are stated next to it.
3. All of the questions are in English. You must answer in English.
4. You must show your working for all questions.
5. This exam contains 8 pages. Check to see if any pages are missing.
6. If you wish to leave before the end of the exam, give your exam script to an invigilator and leave the room quietly. You may not leave in the first 20 minutes, or in the final 10 minutes, of the exam.
7. Switch your mobile phone off and seal it in the envelope provided. Do not open your envelope until the exam is finished or you have left the room.
8. All communication between students, either verbally or non-verbally, is strictly forbidden. Students who finish early must leave the room without communicating with other students.
9. Calculators, mobile phones and any digital means of communication are forbidden. The sharing of pens, erasers or any other item between students is forbidden.
10. All bags, coats, books, notes, etc. must be placed away from your desks and away from the seats next to you. You may not access these during the exam. Take out everything that you will need before the exam starts.
11. Any student found cheating or attempting to cheat will receive a mark of zero (0), and will be investigated according to the regulations of Yükseköğretim Kurumları Öğrenci Disiplin Yönetmeliği.

1. Sınav süresi toplam **120** dakikadır.
2. Soruların her bölümünün kaç puan olduğu yanlarında belirtilmiştir.
3. Tüm sorular İngilizce'dir. Cevaplarınızı İngilizce veriniz.
4. Sonuca ulaşmak için yaptığınız işlemleri ayrıntılarıyla gösteriniz.
5. Sınav 8 sayfadan oluşmaktadır. Lütfen eksik sayfa olup olmadığını kontrol edin.
6. Sınav süresi sona ermeden sınavınızı teslim edip çıkmak isterseniz, sınav kağıdınızı gözetmenlerden birine veriniz ve sınav salonundan sessizce çıkınız. Sınavın ilk 20 dakikası ve son 10 dakikası içinde sınav salonundan çıkmanız yasaktır.
7. Cep telefonunuzu kapatınız ve size verilen zarfın içine koyunuz. Zarfı, sınav süresi bitene kadar ya da sınav salonundan çıkana kadar açmayınız.
8. Sınav esnasında öğrenciler arasında, sözlü ya da sözsüz, her türlü iletişim kesinlikle yasaktır. Sınavını erken bitiren öğrenciler, diğer öğrencilerle hiç bir şekilde iletişim kurmadan sessizce sınıftan çıkmalıdır.
9. Sınav esnasında hesap makinesi, cep telefonu ve dijital bilgi alışverişi yapılan her türlü malzemelerin kullanımı ile diğer silgi, kalem, vb. alışverişlerin yapılması kesinlikle yasaktır.
10. Çanta, palto, kitap ve ders notlarınız gibi eşyalarınız sıraların üzerinden ve yanınızdaki sandalyeden kaldırılmalıdır. Sınav süresince bu tür eşyaları kullanmanız yasaktır, bu nedenle ihtiyacınız olacak herşeyi sınav başlamadan yanınıza alınız.
11. Her türlü sınav, ve diğer çalışmada, kopya çeken veya kopya çekme girişiminde bulunan bir öğrenci, o sınav ya da çalışmadan sıfır (0) not almış sayılır, ve o öğrenci hakkında Yükseköğretim Kurumları Öğrenci Disiplin Yönetmeliği hükümleri uyarınca disiplin kovuşturması yapılır.

α

1	2	3	4	5	6	TOTAL
20	1	20	19	20	20	100



Question 1 (Projections) [20 pts] Find the projection of the point $S(24,-13,11)$ onto the plane $3x-2y+z=11$.

Note first that $\mathbf{n} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ is normal to the plane and that $P(0,0,11)$ is a point in the plane.
We calculate that

$$\overrightarrow{SP} = P - S = (0,0,11) - (24,-13,11) = (-24,13,0)$$

and that

$$\begin{aligned}\text{proj}_{\mathbf{n}} \overrightarrow{SP} &= \left(\frac{\overrightarrow{SP}\cdot\mathbf{n}}{\|\mathbf{n}\|^2}\right)\mathbf{n} = \left(\frac{-72-26+0}{9+4+1}\right)(3\mathbf{i}-2\mathbf{j}+\mathbf{k}) \\ &= \left(\frac{-98}{14}\right)(3\mathbf{i}-2\mathbf{j}+\mathbf{k}) = -7(3\mathbf{i}-2\mathbf{j}+\mathbf{k}) = -21\mathbf{i}+14\mathbf{j}-7\mathbf{k}.\end{aligned}$$

Therefore

$$\text{proj}_{\textbf{plane}}\, S = S + \text{proj}_{\mathbf{n}} \overrightarrow{SP} = (24,-13,11) + (-21,14,-7) = (3,1,4).$$

$$\text{proj}_{\textbf{plane}}\, S = \left(\square\, , \square\, , \square \right)$$

Question 2 [1 pt] Please write your student number at the top-right of this page.

STUDENT NO:

Question 3 (The Fundamental Theorem of Calculus)

(a) [10 pts] Calculate $\dfrac{d}{dx}\displaystyle\int_2^{e^x} \frac{1}{\ln t}\, dt$.

Let $u = e^x$. By the Chain Rule and the Fundamental Theorem of Calculus, it follows that

$$\begin{aligned}\frac{d}{dx}\int_2^{e^x} \frac{1}{\ln t}\, dt &= \left(\frac{d}{du}\int_2^{u} \frac{1}{\ln t}\, dt\right)\left(\frac{du}{dx}\right) \\ &= \left(\frac{1}{\ln u}\right)(e^x) = \frac{e^x}{\ln e^x} = \frac{e^x}{x}.\end{aligned}$$

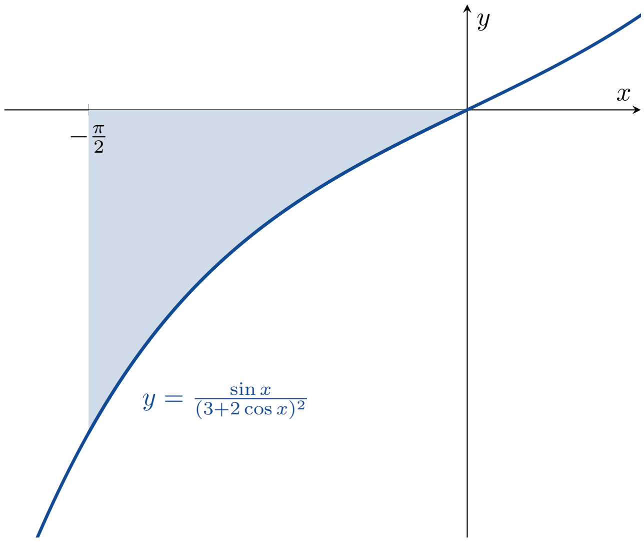
$$\frac{d}{dx}\int_2^{e^x} \frac{1}{\ln t}\, dt = \boxed{}$$

(b) [10 pts] Calculate $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \frac{\sin 2\theta}{2\sin\theta}\, d\theta$.

We calculate that

$$\int_{\frac{\pi}{2}}^{\pi} \frac{\sin 2\theta}{2\sin\theta}\, d\theta = \int_{\frac{\pi}{2}}^{\pi} \frac{2\sin\theta\cos\theta}{2\sin\theta}\, d\theta = \int_{\frac{\pi}{2}}^{\pi} \cos\theta\, d\theta = [\sin\theta]_{\frac{\pi}{2}}^{\pi} = \sin\pi - \sin\frac{\pi}{2} = 0 - 1 = -1.$$

$$\int_{\frac{\pi}{2}}^{\pi} \frac{\sin 2\theta}{2\sin\theta}\, d\theta = \boxed{}$$



Question 4 (The Substitution Method) [19 pts] Calculate $\displaystyle\int_{-\frac{\pi}{2}}^{0} \frac{\sin x}{(3+2\cos x)^2}\,dx$.

Let $u = 3 + 2\cos x$. Then $du = -2\sin x\,dx$. Moreover $x = -\frac{\pi}{2} \implies u = 3 + 2\cos -\frac{\pi}{2} = 3$ and $x = 0 \implies u = 3 + 2\cos 0 = 5$.
Therefore

$$\int_{-\frac{\pi}{2}}^{0} \frac{\sin x}{(3+2\cos x)^2}\,dx = \int_3^5 -\frac{1}{2u^2}\,du = \int_3^5 -\frac{1}{2}u^{-2}\,du = \left[\frac{1}{2}u^{-1}\right]_3^5 = \frac{1}{10} - \frac{1}{6} = -\frac{1}{15}.$$

$$\int_{-\frac{\pi}{2}}^{0} \frac{\sin x}{(3+2\cos x)^2}\,dx = \boxed{}$$

Question 5 (Lines) The following two lines do not intersect:

line 1: $x = 1 - 3t$, $y = 10$, $z = -2 + t$

line 2: $x = 19 + 6s$, $y = 20$, $z = -8 - 2s$

(a) [1 pt] Find a vector $\mathbf{v}_1$ which is parallel to **line 1**.

$$\mathbf{v}_1 = \square\,\mathbf{i}\ \square\,\mathbf{j}\ \square\,\mathbf{k}$$

(b) [1 pt] Find a vector $\mathbf{v}_2$ which is parallel to **line 2**.

$$\mathbf{v}_2 = \square\,\mathbf{i}\ \square\,\mathbf{j}\ \square\,\mathbf{k}$$

(c) [4 pts] Calculate $\mathbf{v}_1 \times \mathbf{v}_2$.

(d) [1 pt] Are these two lines **parallel** to each other?

(e) [13 pts] Find the **distance** between these two lines.

Clearly $\mathbf{v}_1 = -3\mathbf{i} + \mathbf{k}$ and $\mathbf{v}_2 = 6\mathbf{i} - 2\mathbf{k} = -2\mathbf{v}_1$. Thus the lines are parallel and $\mathbf{v}_1 \times \mathbf{v}_2 = \mathbf{0}$. Hence we must use the formula

$$d = \frac{\left\|\overrightarrow{P_1P_2} \times \mathbf{v}_1\right\|}{\|\mathbf{v}_1\|}.$$

We can see that the point $P_1(1, 10, -2)$ lies on line 1, and that the point $P_2(19, 20, -8)$ lies on line 2. Then we calculate that

$$\begin{aligned}\overrightarrow{P_1P_2} &= P_2 - P_1 = (19, 20, -8) - (1, 10, -2)\\ &= (18, 10, -6)\\ &= 18\mathbf{i} + 10\mathbf{j} - 6\mathbf{k}\end{aligned}$$

and

$$\begin{aligned}\overrightarrow{P_1P_2} \times \mathbf{v}_1 &= (18, 10, -6) \times (-3, 0, 1)\\ &= (10, 0, 30)\\ &= 10\mathbf{i} + 30\mathbf{k}.\end{aligned}$$

Therefore

$$\begin{aligned}d &= \frac{\left\|\overrightarrow{P_1P_2} \times \mathbf{v}_1\right\|}{\|\mathbf{v}_1\|}\\ &= \frac{\|10\mathbf{i} + 30\mathbf{k}\|}{\|-3\mathbf{i} + \mathbf{k}\|}\\ &= \frac{\sqrt{100 + 0 + 900}}{\sqrt{9 + 0 + 1}}\\ &= \frac{\sqrt{1000}}{\sqrt{10}}\\ &= 10.\end{aligned}$$

$$d = \square$$

Question 6 (Spherical Polar Coordinates in $\mathbb{R}^3$/ Polar Coordinates in $\mathbb{R}^2$)

(a) [10 pts] Convert the spherical polar coordinates $(\rho, \theta, \phi) = (\sqrt{3}, -60°, 30°)$ into Cartesian coordinates.

We calculate that

$$x = \rho \sin\phi \cos\theta = \sqrt{3}\sin 30° \cos -60° = \sqrt{3}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{4},$$

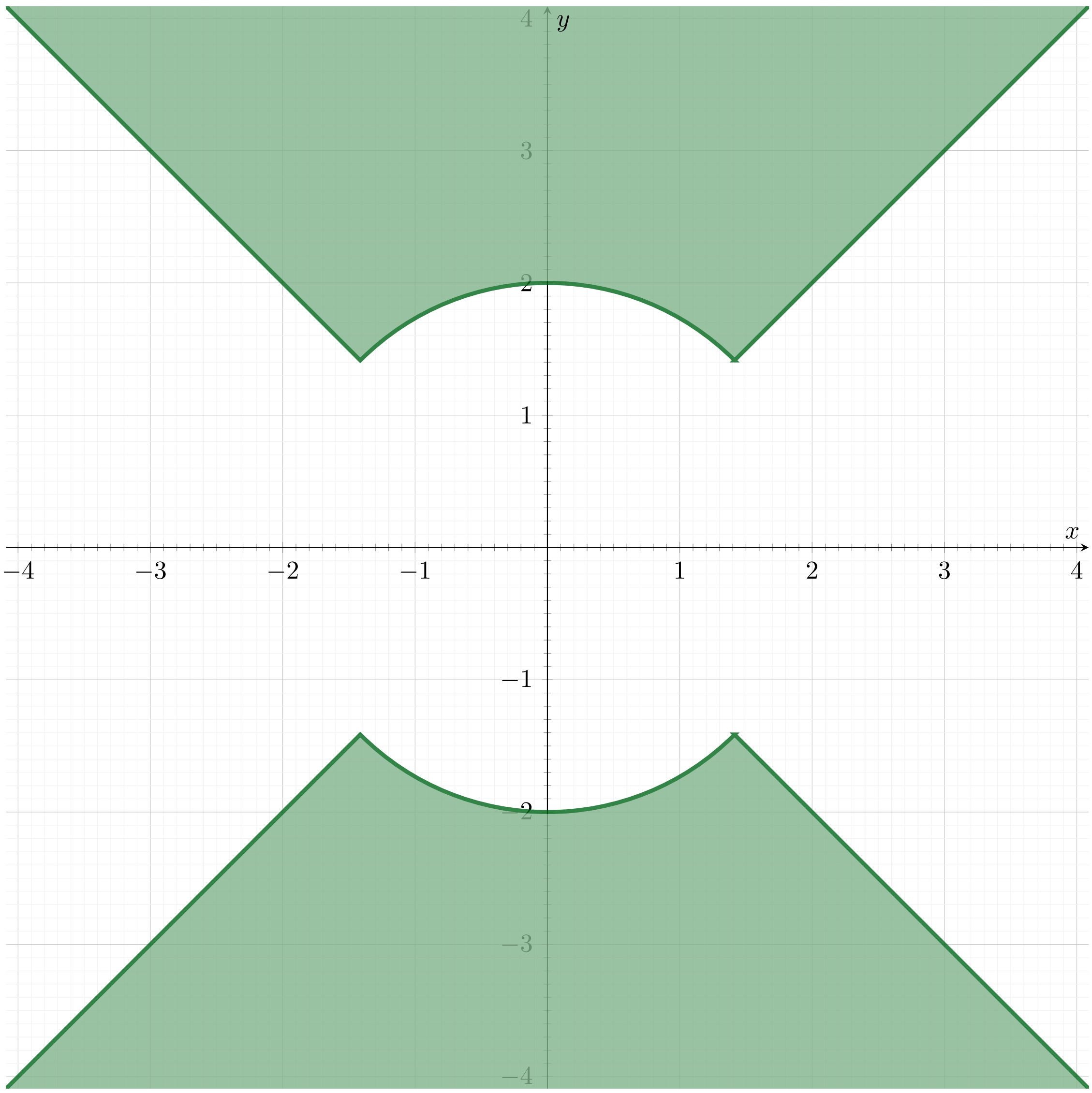
$$y = \rho \sin\phi \sin\theta = \sqrt{3}\sin 30° \sin -60° = \sqrt{3}\left(\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) = -\frac{3}{4},$$

$$z = \rho \cos\phi = \sqrt{3}\cos 30° = \sqrt{3}\left(\frac{\sqrt{3}}{2}\right) = \frac{3}{2}.$$

Hence the point $(\rho, \theta, \phi) = (2, 30°, 60°)$ has Cartesian coordinates $(x, y, z) = \left(\frac{\sqrt{3}}{4}, -\frac{3}{4}, \frac{3}{2}\right)$.

$$(x, y, z) = \left(\square \, , \, \square \, , \, \square \right)$$

(b) [10 pts] Draw the set of points whose polar coordinates satisfy $|r| \geq 2$ and $45° \leq \theta \leq 135°$.



this page is empty

$$\cos\theta = \sin\left(\tfrac{\pi}{2}-\theta\right)$$
$$\cos^2\theta + \sin^2\theta = 1$$
$$1+\tan^2\theta = \sec^2\theta$$
$$1+\cot^2\theta = \operatorname{cosec}^2\theta$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos^2\theta = \tfrac{1}{2}(1+\cos 2\theta)$$
$$\sin^2\theta = \tfrac{1}{2}(1-\cos 2\theta)$$

$$x = r\cos\theta \qquad y = r\sin\theta \qquad x^2+y^2 = r^2$$

$$x = \rho\sin\phi\cos\theta \qquad y = \rho\sin\phi\sin\theta \qquad z = \rho\cos\phi \qquad \rho = \sqrt{x^2+y^2+z^2}$$

$$\cos 0 = \cos 0^\circ = 1 \qquad \sin 0 = \sin 0^\circ = 0$$
$$\cos\tfrac{\pi}{6} = \cos 30^\circ = \tfrac{\sqrt{3}}{2} \qquad \sin\tfrac{\pi}{6} = \sin 30^\circ = \tfrac{1}{2}$$
$$\cos\tfrac{\pi}{4} = \cos 45^\circ = \tfrac{1}{\sqrt{2}} \qquad \sin\tfrac{\pi}{4} = \sin 45^\circ = \tfrac{1}{\sqrt{2}}$$
$$\cos\tfrac{\pi}{3} = \cos 60^\circ = \tfrac{1}{2} \qquad \sin\tfrac{\pi}{3} = \sin 60^\circ = \tfrac{\sqrt{3}}{2}$$
$$\cos\tfrac{\pi}{2} = \cos 90^\circ = 0 \qquad \sin\tfrac{\pi}{2} = \sin 90^\circ = 1$$

$$(uv)' = uv' + u'v$$
$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$
$$(f\circ g)'(x) = f'\left(g(x)\right)g'(x)$$

$$\tfrac{d}{dx}x^n = nx^{n-1}$$
$$\tfrac{d}{dx}\sin x = \cos x$$
$$\tfrac{d}{dx}\cos x = -\sin x$$
$$\tan x = \tfrac{\sin x}{\cos x} \qquad \tfrac{d}{dx}\tan x = \sec^2 x$$
$$\sec x = \tfrac{1}{\cos x} \qquad \tfrac{d}{dx}\sec x = \sec x\tan x$$
$$\cot x = \tfrac{\cos x}{\sin x} \qquad \tfrac{d}{dx}\cot x = -\operatorname{cosec}^2 x$$
$$\operatorname{cosec} x = \tfrac{1}{\sin x} \qquad \tfrac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x\cot x$$
$$\tfrac{d}{dx}e^x = e^x$$
$$\tfrac{d}{dx}\ln|x| = \tfrac{1}{x}$$

$$\operatorname{av}(f) = \frac{1}{b-a}\int_a^b f(x)\,dx$$
$$V = \int_a^b A(x)\,dx \qquad V = \int_a^b \pi\left(R(x)\right)^2\,dx$$

$$c = \sqrt{a^2-b^2} \qquad \text{or} \qquad c = \sqrt{a^2+b^2}$$

$$\operatorname{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} \qquad \theta = \cos^{-1}\left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}\right)$$
$$\mathbf{u}\times\mathbf{v} = (u_2v_3-u_3v_2)\mathbf{i} - (u_1v_3-u_3v_1)\mathbf{j} + (u_1v_2-u_2v_1)\mathbf{k}$$
$$d = \frac{\left\|\overrightarrow{PS}\times\mathbf{v}\right\|}{\|\mathbf{v}\|} \qquad d = \frac{\left|\overrightarrow{PS}\cdot\mathbf{n}\right|}{\|\mathbf{n}\|}$$
$$d = \frac{\left\|\overrightarrow{P_1P_2}\times\mathbf{v}_1\right\|}{\|\mathbf{v}_1\|} \qquad d = \frac{\left|\overrightarrow{P_1P_2}\cdot(\mathbf{v}_1\times\mathbf{v}_2)\right|}{\|\mathbf{v}_1\times\mathbf{v}_2\|}$$