Your Name / Adınız - Soyadınız	Your Signature / İmza

Student ID # / Öğrenci No

Professor's Name / Öğretim Üyesi	Your Department / Bölüm

- Calculators, cell phones off and away!.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. **Show your work in evaluating any limits, derivatives**.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- Use a **BLUE ball-point pen** to fill the cover sheet. Please make sure that your exam is complete.
- Time limit is 80 min.

Do not write in the table to the right.

Problem	Points	Score
1	18	
2	22	
3	20	
4	20	
5	20	
Total:	100	

1. (a) $\boxed{\text{10 Points}}$ Find the value of $\displaystyle\int_{-\pi/2}^{\pi/2}\left(2+\tan\left(\frac{x}{2}\right)\right)\sec^2\left(\frac{x}{2}\right)\,dx$.

Solution: Let $u=2+\tan\left(\frac{x}{2}\right)$ and so $du=\frac{1}{2}\sec^2\left(\frac{x}{2}\right)\,dx$. When $x=-\pi/2$, we have $u=2+\tan\left(-\frac{\pi}{4}\right)=2-1=1$ and when $x=\pi/2$, we have $u=2+\tan\left(\frac{\pi}{4}\right)=2+1=3$. Therefore

$$\begin{aligned}\int_{-\pi/2}^{\pi/2}\left(2+\tan\left(\frac{x}{2}\right)\right)\sec^2\left(\frac{x}{2}\right)\,dx &= 2\int_{-\pi/2}^{\pi/2}\left(2+\tan\left(\frac{x}{2}\right)\right)\frac{1}{2}\sec^2\left(\frac{x}{2}\right)\,dx\\ &= 2\int_1^3 u\,du\\ &= 2\left[\frac{1}{2}u^2\right]_1^3=(9-1)=\boxed{8}\end{aligned}$$

p.317, pr.14(b)

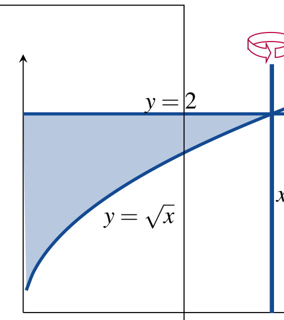
(b) $\boxed{\text{8 Points}}$ For $|x|<\pi$, let $y=\displaystyle\int_0^{\cos x}\frac{dt}{\sqrt{1-t^2}}$. Find dy/dx.

Solution: Let $u=\cos x$. Then since $\pi/2<x<\pi$, we have $|\sin x|=\sin x$ and so by the Fundamental Theorem of Calculus,

$$\frac{dy}{dx}=\frac{d}{dx}\int_0^{\cos x}\frac{dt}{\sqrt{1-t^2}}=\frac{d}{dx}\int_0^{u}\frac{dt}{\sqrt{1-t^2}}=\frac{1}{\sqrt{1-u^2}}\cdot\frac{du}{dx}=\frac{1}{\sqrt{1-u^2}}\cdot(-\sin x)=\frac{-\sin x}{\sqrt{1-(\cos x)^2}}=\frac{-\sin x}{\sqrt{\sin^2 x}}=\frac{-\sin x}{|\sin x|}=\boxed{-1}$$

p.241, pr.65(a)

2. (a) $\boxed{\text{12 Points}}$ Find the *volume* of solid generated by revolving the region bounded by $y=\sqrt{x}$, $y=2$, and $x=0$ about the line $x=4$.

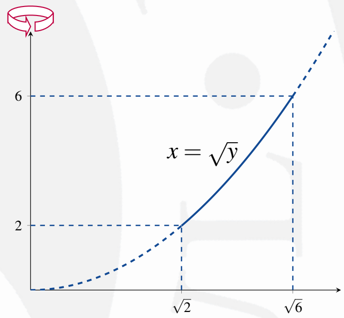


Solution:
If $0 \le x \le 4$, then a vertical strip of the given region "at" x has length $2-\sqrt{x}$ and moves around a circle of radius $4-x$, so the volume generated rotation of that region around x-axis is

$$V = \int_a^b 2\pi \left(\begin{array}{c}\text{shell}\\ \text{radius}\end{array}\right)\left(\begin{array}{c}\text{shell}\\ \text{height}\end{array}\right)\,dx = \int_0^4 2\pi(4-x)(2-\sqrt{x})\,dx$$
$$= 2\pi\int_0^4 (8-4x^{1/2}-2x+x^{3/2})\,dx$$
$$= 2\pi\left[8x-\frac{8}{3}-x^2+\frac{2}{5}x^{5/2}\right]_0^4 = 2\pi(32-\frac{64}{3}-16+\frac{64}{5}) = \frac{2\pi}{15}(240-320+192) = \frac{224\pi}{15}.$$

p.345, pr.32(c)

(b) $\boxed{\text{10 Points}}$ Find the *area of surface* generated by revolving the curve $x=\sqrt{y}$, $2 \le y \le 6$, about y-axis.



Solution: The surface area formula we shall use is $S = \int_c^d 2\pi x\sqrt{1+(dx/dy)^2}\,dy$

$$x=\sqrt{y} \Rightarrow \frac{dx}{dy} = \frac{1}{2\sqrt{y}} \Rightarrow \left(\frac{dx}{dy}\right)^2 = \frac{1}{4y} \Rightarrow S = \int_2^6 2\pi(\sqrt{y})\sqrt{1+\frac{1}{4y}}\,dy$$
$$S = \cancel{2}\pi\int_2^6 \cancel{\sqrt{y}}\ \frac{\sqrt{4y+1}}{\cancel{2}\cancel{\sqrt{y}}}\,dy = \pi\int_2^6 \sqrt{4y+1}\,dy \qquad \boxed{u=4y+1, \qquad du = 4\,dy \Rightarrow}$$
$$= \frac{\pi}{4}\int_9^{25}\sqrt{u}\,du = \frac{\pi}{4}\left[\frac{u^{3/2}}{3/2}\right]_9^{25} = \frac{\pi}{6}\left[(25)^{3/2}-(9)^{3/2}\right]$$
$$= \frac{\pi}{6}[125-27]$$
$$\longrightarrow S = \boxed{\frac{49\pi}{3}}$$

p.377, pr.24

3. (a) $\boxed{\text{10 Points}}$ For what values of a and b is

$$f(x) = \begin{cases} -2 & x \le -1 \\ ax-b, & -1 < x < 1 \\ 3, & x \ge 1 \end{cases}$$

continuous at every x. p.83, pr.45

Solution: Clearly f is continuous if $x \neq -1$ and for $x \neq 1$ for if $x < -1$ or if $-1 < x < 1$ or if $x > 1$, f is a polynomial, regardless the values of a and b. For continuity at $x = -1$, we require that the one-sided limits of $f(x)$ at $x = -1$ be equal. But $\lim_{x\to -1^-} f(x) = -2$ and $\lim_{x\to -1^+} f(x) = a(-1) - b = -a - b$.
Similarly, for continuity at $x = 1$, we require that the one-sided limits of $f(x)$ at $x = 1$ be equal. But $\lim_{x\to 1^-} f(x) = a(1) - b = a - b$ and $\lim_{x\to 1^+} f(x) = 3$.
Equality of one-sided limits is equivalent to

$$-2 = -a - b \text{ and } a - b = 3$$
$$\Longrightarrow a = -\frac{5}{2} \text{ and } b = -\frac{1}{2}.$$

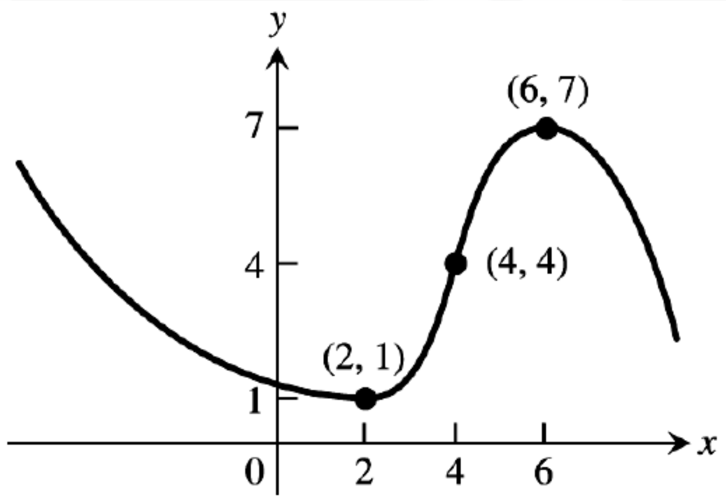
p.583, pr.32

(b) $\boxed{\text{10 Points}}$ Using the following properties of a twice-differentiable function $y = f(x)$, sketch its graph on the grid by indicating all significant points.

Solution: The given table indicates that the graph

- is decreasing and concave up on $(-\infty, 2)$
- has a local minimum at $(2, 1)$
- increasing and concave up on $(2, 4)$
- has a point of inflection at $(4, 4)$
- increasing and concave down on $(4, 6)$
- has a local maximum at $(6, 7)$
- is decreasing and concave down on $(6, +\infty)$

x	y	**Derivatives**	
$x < 2$		$y' < 0$,	$y'' > 0$
2	1	$y' = 0$,	$y'' > 0$
$2 < x < 4$		$y' > 0$,	$y'' > 0$
4	4	$y' > 0$,	$y'' = 0$
$4 < x < 6$		$y' > 0$,	$y'' < 0$
6	7	$y' = 0$,	$y'' < 0$
$x > 6$		$y' < 0$,	$y'' < 0$



p.452, pr.24

4. (a) $\boxed{\text{10 Points}}$ Find the limit $\lim\limits_{x\to 0^+} \dfrac{\ln(e^x-1)}{\ln x}$.

Solution: As $x \to 0^+$, $e^x - 1$ is positive and approaches 0 and so the numerator approaches $-\infty$. Likewise the denominator approaches $-\infty$. Thus this limit has the indeterminate $\left(\frac{\infty}{\infty}\right)$. Hence the L'Hôpital's Rule applies. We have

$$\begin{aligned}
\lim_{x\to 0^+} \frac{\ln(e^x-1)}{\ln x} &= \lim_{x\to 0^+} \frac{\frac{e^x}{e^x-1}}{\frac{1}{x}} \\
&= \lim_{x\to 0^+} \frac{xe^x}{e^x-1} = \lim_{x\to 0^+} \frac{e^x+xe^x}{e^x} \\
&= \frac{e^0+0e^0}{e^0} = \boxed{1}
\end{aligned}$$

p.423, pr.34

(b) $\boxed{\text{10 Points}}$ Suppose $f(x) = (2x^3+1)^{1/5}$. Find $f^{-1}(x)$. Write the domain and the range of f^{-1}. As a check, show *only* that $f(f^{-1})(x) = x$.

Solution: Let $y = (2x^3+1)^{1/5}$. Then $y^5 = 2x^3+1$ and so $y^5 - 1 = 2x^3$. Therefore $\sqrt[3]{\dfrac{y^5-1}{2}} = x$. Hence

$$\boxed{f^{-1}(x) = \sqrt[3]{\frac{x^5-1}{2}}}$$

Domain of $f^{-1}(x)$ is: $\boxed{(-\infty,+\infty)}$ ____________________

Range of $f^{-1}(x)$ is: $\boxed{(-\infty,+\infty)}$ ____________________

$$f(f^{-1}(x)) = f\left(\sqrt[3]{\frac{x^5-1}{2}}\right) = (2\left(\sqrt[3]{\frac{x^5-1}{2}}\right)^3 + 1)^{1/5} = \left(\left(x^5-1\right)+1\right)^{1/5} = (x^5)^{1/5} = x$$

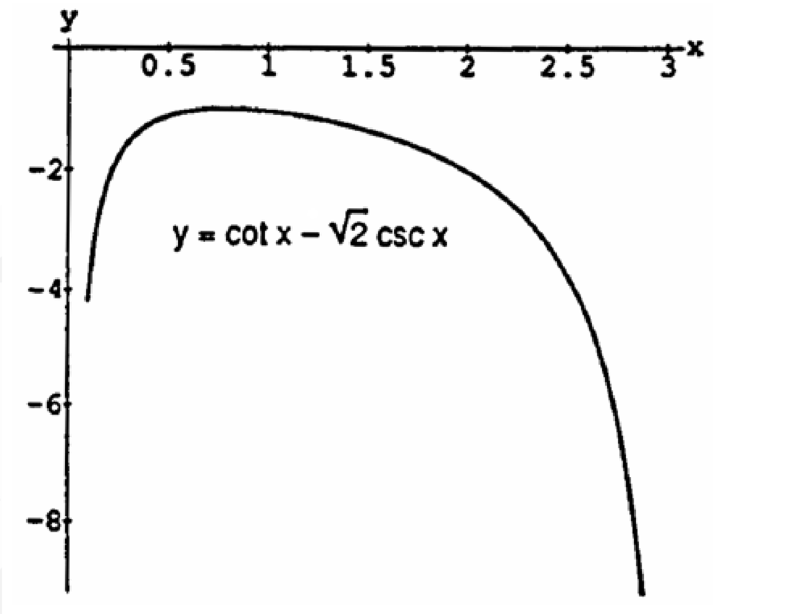
p.387, pr.32

5. (a) $\boxed{\text{8 Points}}$ The function $y = \cot x - \sqrt{2}\csc x$ has an absolute maximum value on the interval $0 < x < \pi$. Find it.

Solution: If $y = \cot x - \sqrt{2}\csc x$ where $0 < x < \pi$, then $y' = -(\csc x)(\csc x - \sqrt{2}\cot x)$. Solving

$y' = 0 \Rightarrow \cos x = \frac{1}{\sqrt{2}} \Rightarrow x = \frac{\pi}{4}$. For $0 < x < \frac{\pi}{4}$, we have $y' > 0$, and $y' < 0$ when $\frac{\pi}{4} < x < \pi$.

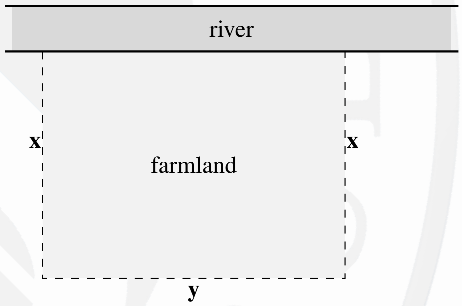
Therefore at $x = \dfrac{\pi}{4}$ there is a maximum value of $y = -1$.



The graph confirms the findings in (a).

p.241, pr.65(a)

(b) 12 Points We want to fence in a rectangular piece of farmland next to a river. We will use the river as one edge of the rectangle. We have 800m of fence to use for the other three edges. What are dimensions of the largest area we can enclose?



Solution: We're going to enclose three sides– two of dimension x (perpendicular to the river) and one of $800-2x$ (the amount of fence remaining after fencing the other two sides)

$2x+y=800$

$A=xy$ area of farmland

$y=800-2x$

$A=800x-2x^2$

$A'(x)=800-4x$ derivative of area with respect to x

$0=800-4x$ find out where f' equals 0

$4x=800$

$x=200$

$A'(x)=800-4x$

$A''(x)=-4$ check 2nd derivative. if negative then local max at $x=200$

$2x + y = 800$

$y = 400$
dimensions 100×200

p.236, pr.7