Cep telefonunuzu gözetmene teslim ediniz / Deposit your cell phones to invigilator

Your Name / Adınız - Soyadınız

Your Signature / İmza

Student ID # / Öğrenci No

Professor's Name / Öğretim Üyesi

Your Department / Bölüm

- Calculators, cell phones off and away!.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. **Show your work in evaluating any limits, derivatives**.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- Use a **BLUE ball-point pen** to fill the cover sheet. Please make sure that your exam is complete.
- Time limit is 90 min.

Do not write in the table to the right.

Problem	Points	Score
1	24	
2	25	
3	27	
4	24	
Total:	100	

1. (a) $\boxed{\text{12 Points}}$ If it exists, find the limit $\lim\limits_{x\to 0}\dfrac{2^{\sin x}-1}{e^x-1}$

Solution: The limit leads to the indeterminate $\dfrac{0}{0}$. Hence using L'Hôpital's Rule, we have

$$\lim_{x\to 0}\frac{2^{\sin x}-1}{e^x-1}\underset{\text{L'H}}{\overset{\left[\frac{0}{0}\right]}{=}}\lim_{x\to 0}\frac{2^{\sin x}\cos x(\ln 2)}{e^x}=\frac{2^{\sin 0}\cos 0(\ln 2)}{e^0}=\frac{2^0(1)(\ln 2)}{1}=\boxed{\ln 2}$$

p.652, pr.3

(b) $\boxed{\text{12 Points}}$ Evaluate the definite integral $\displaystyle\int_0^{\ln 9} e^\theta(e^\theta-1)^{1/2}\,d\theta$.

Solution: Let $u=e^\theta-1$. Then $du=e^\theta\,d\theta$. When $\theta=0$, we have $u=e^0-1=1-1=0$ and when $\theta=\ln 9$, we have $u=e^{\ln 9}-1=9-1=8$. Hence

$$\int_0^{\ln 9} e^\theta(e^\theta-1)^{1/2}\,d\theta=\int_0^{\ln 9}\underbrace{(e^\theta-1)^{1/2}}_{u^{1/2}}\underbrace{e^\theta\,d\theta}_{du}$$
$$=\int_0^8 u^{1/2}\,du=\left[\frac{u^{1/2+1}}{1/2+1}\right]_0^8=\frac{2}{3}(8^{3/2}-0^{3/2})=\frac{2}{3}(2^{9/2}-0)=\frac{2^{11/2}}{3}=\boxed{\frac{32\sqrt{2}}{3}}$$

p.652, pr.3

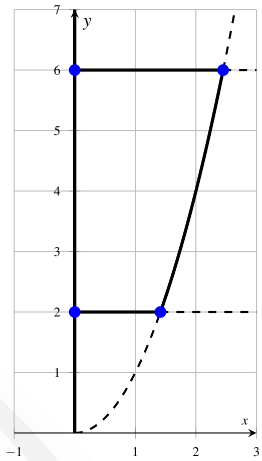
2. (a) $\boxed{\text{12 Points}}$ If $f(x)=2x^3+3x+1$, then find the value of $\dfrac{df^{-1}}{dx}$ at $x=6=f(1)$.

Solution:

$$\frac{df}{dx}=6x^2+3\Rightarrow\left[\frac{df^{-1}}{dx}\right]_{x=6}=\left[\frac{1}{\frac{df}{dx}}\right]_{x=1}=\left[\frac{1}{6x^2+3}\right]_{x=1}=\boxed{\frac{1}{9}}$$

p.83, pr.40

(b) $\boxed{\text{13 Points}}$ Find the area of the surface generated by revolving the curve $x = \sqrt{y}$, $2 \le y \le 6$, about y-axis.



Solution: The surface area formula we shall use is $S = \int_c^d 2\pi x\sqrt{1+(dx/dy)^2}\,\mathrm{d}y$

$$x = \sqrt{y} \Rightarrow \frac{dx}{dy} = \frac{1}{2\sqrt{y}} \Rightarrow \left(\frac{dx}{dy}\right)^2 = \frac{1}{4y} \Rightarrow S = \int_2^6 2\pi(\sqrt{y})\sqrt{1+\frac{1}{4y}}\,dy$$

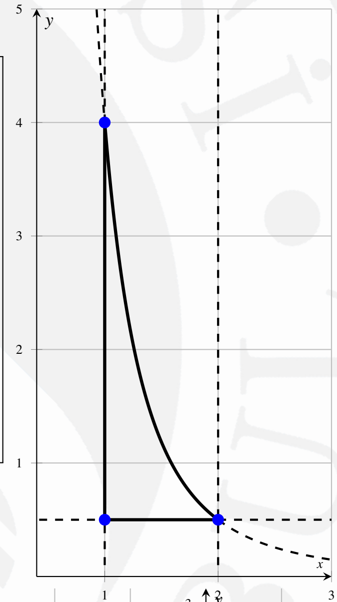
$$S = \cancel{2}\pi\int_2^6 \cancel{\sqrt{y}}\,\frac{\sqrt{4y+1}}{\cancel{2}\cancel{\sqrt{y}}}\,dy = \pi\int_2^6 \sqrt{4y+1}\,\mathrm{d}y \qquad \boxed{u = 4y+1, \qquad du = 4\,dy \Rightarrow}$$

$$= \frac{\pi}{4}\int_9^{25}\sqrt{u}\,du = \frac{\pi}{4}\left[\frac{u^{3/2}}{3/2}\right]_9^{25} = \frac{\pi}{6}\left[(25)^{3/2}-(9)^{3/2}\right] = \frac{\pi}{6}[125-27]$$

$$\longrightarrow S = \boxed{\frac{49\pi}{3}}$$

4.4, pr.102

3. (a) $\boxed{\text{13 Points}}$ Find the volume of the solid generated by revolving the region bounded by $y = \dfrac{4}{x^3}$ and the lines $x = 1$, $y = 1/2$ about the line $x = 2$.



Solution:

$$V = \int_a^b 2\pi\left(\begin{array}{c}\text{shell}\\ \text{radius}\end{array}\right)\left(\begin{array}{c}\text{shell}\\ \text{height}\end{array}\right)\,\mathrm{d}x$$

$$= \int_1^2 2\pi(2-x)\left(\frac{4}{x^3}-\frac{1}{2}\right)\,\mathrm{d}x$$

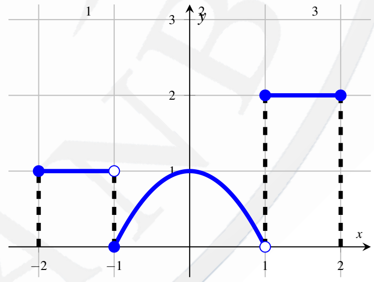
$$= 2\pi\int_1^2\left(\frac{8}{x^3}-\frac{4}{x^2}-1+\frac{x}{2}\right)\,\mathrm{d}x = 2\pi\left[-\frac{4}{x^2}+\frac{4}{x}-x+\frac{x^2}{4}\right]_1^2$$

$$= 2\pi((-1+2-2+1)-(-4+4-1+\frac{1}{4})) = \boxed{\frac{3\pi}{2}}.$$

p.879, pr.42

(b) $\boxed{\text{14 Points}}$ Suppose

$$u(x) = \begin{cases} 1, & -2 \le x < -1 \\ 1-x^2 & -1 \le x < 1 \\ 2, & 1 \le x \le 2. \end{cases}$$



Graph this function and find the integral $\int_{-2}^{2} u(x)\,dx$.

Solution:

$$\int_{-2}^{2} u(x)\,dx = \int_{-2}^{-1} dx + \int_{-1}^{1}(1-x^2)\,dx + \int_1^2 2\,dx$$

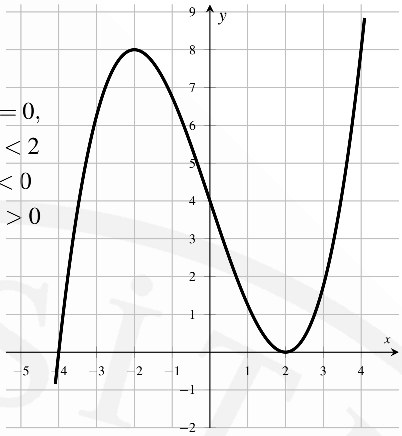
$$= [x]_{-2}^{-1} + \left[x-\frac{1}{3}x^3\right]_{-1}^{1} + [2x]_1^2 = (-1-(-2)) + \left[(1-\frac{1^3}{3})-(-1-\frac{(-1)^3}{3})\right]$$

$$+[2(2)-2(1)] = 1+\frac{2}{3}-\left(-\frac{2}{3}+4-2\right) = \boxed{\frac{13}{3}}$$

p.112, pr.26

4. (a) $\boxed{\text{12 Points}}$ Sketch the graph of a function f that has the following properties:

$$\begin{cases} f \text{ is everywhere continuous} & \\ f(-2) = 8, & f'(2) = f'(-2) = 0, \\ f(0) = 4, & f'(x) < 0 \text{ for } |x| < 2 \\ f(2) = 0, & f''(x) < 0 \text{ for } x < 0 \\ f'(x) > 0 \text{ for } |x| > 2 & f''(x) > 0, \text{ for } x > 0 \end{cases}$$



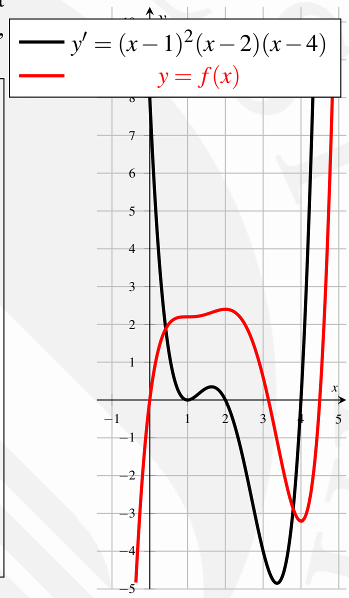
Solution: One such example is

$$f(x) = x^3/4 - 3x + 4.$$

It is easy to check that this function fulfills all requirements.

p.241, pr.45

(b) $\boxed{\text{12 Points}}$ Suppose the derivative of $y = f(x)$ is $y' = (x-1)^2(x-2)(x-4)$. At what points, if any, does the graph of f have a local minimum, local maximum, or point of inflection?



Solution: When $y' = (x-1)^2(x-2)(x-4)$, then

$$\begin{aligned} y'' &= 2(x-1)(x-2)(x-4) + (x-1)^2(x-4) + (x-1)^2(x-2) \\ &= (x-1)\left[2(x^2-6x+8) + (x^2-5+4) + (x^2-3x+2)\right] \\ &= 2(x-1)(2x^2-10x+11). \end{aligned}$$

The curve rises on $(-\infty, 2)$ and $(4, \infty)$ and falls on $(2, 4)$. At $x = 2$, there is a local maximum and at $x = 4$ a local minimum. The curve is concave downward on $(-\infty, 1)$ and $\left(\frac{5-\sqrt{3}}{2}, \frac{5+\sqrt{3}}{2}\right)$ and concave upward on $\left(1, \frac{5-\sqrt{3}}{2}\right)$ and $\left(\frac{5+\sqrt{3}}{2}, \infty\right)$. At $x = 1, x = \frac{5-\sqrt{3}}{2}, x = \frac{5+\sqrt{3}}{2}$ there are inflection points.

p.212, pr.85