Your Name / Adınız - Soyadınız

Your Signature / İmza

Student ID # / Öğrenci No

Professor's Name / Öğretim Üyesi

Your Department / Bölüm

- Calculators, cell phones off and away!.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. **Show your work in evaluating any limits, derivatives**.
- Place a box around your answer to each question.
- Use a **BLUE ball-point pen** to fill the cover sheet. Please make sure that your exam is complete.
- Time limit is 75 min.

Do not write in the table to the right.

Problem	Points	Score
1	22	
2	25	
3	27	
4	26	
Total:	100	

1. (a) 10 Points Find, if it exists, the limit $\lim_{x\to 2}\frac{\sin(x^2-4)}{x-2}$ (**You are not allowed to use the L'Hôpital's Rule**).

Solution: Making the substitution $t = x^2-4$ and noting that $t\to 0$ as $x\to 2$, we have $\lim_{x\to 2}\frac{\sin(x^2-4)}{x-2} = \lim_{t\to 0}\frac{\sin(t)}{t} = 1$. Therefore

$$\begin{aligned}\lim_{x\to 2}\frac{\sin(x^2-4)}{x-2} &= \lim_{x\to 2}\frac{\sin(x^2-4)}{x^2-4}(x+2)\\ &= \lim_{x\to 2}\frac{\sin(x^2-4)}{x-2}\lim_{x\to 2}(x+2)\\ &= (1)(2+2) = \boxed{4}\end{aligned}$$

p.652, pr.3

(b) 12 Points Suppose $f(x)=\sqrt{4-x}$, $x_0=0$, $L=2$. Find a $\delta>0$ such that for all x with $0<|x-x_0|<\delta$, the inequality $|f(x)-L|<\varepsilon$ holds.

Solution: First, we want:

$$\begin{aligned}|\sqrt{4-x}-2|<\varepsilon &\Rightarrow -\varepsilon<\sqrt{4-x}-2<\varepsilon \Rightarrow 2-\varepsilon<\sqrt{4-x}<2+\varepsilon\\ &\Rightarrow (2-\varepsilon)^2<4-x<(2+\varepsilon)^2\\ &\Rightarrow -(2+\varepsilon)^2<x-4<-(2-\varepsilon)^2 \Rightarrow 4-(2+\varepsilon)^2<x<4-(2-\varepsilon)^2\end{aligned}$$

Moreover

$$|x-0| < \delta \Rightarrow -\delta < x < \delta.$$

Then $-\delta = 4-(2+\varepsilon)^2 = \cancel{4} - \cancel{4} - 4\varepsilon - \varepsilon^2 = -4\varepsilon - \varepsilon^2 \Rightarrow \delta = 4\varepsilon + \varepsilon^2$ or $\delta = 4-(2-\varepsilon)^2 = 4\varepsilon - \varepsilon^2$; thus, we choose the smaller distance $\delta = \min\{4\varepsilon+\varepsilon^2, 4\varepsilon-\varepsilon^2\} = \boxed{4\varepsilon - \varepsilon^2}$.

p.573, pr.18

2. (a) $\boxed{\text{15 Points}}$ For what value(s) of a and b is

$$f(x) = \begin{cases} ax+b & x \le 0 \\ x^2+3a-b & 0 < x \le 2 \\ 3x-5, & x > 2 \end{cases}$$

is continuous at every x.

Solution: Since each piece is a polynomial and polynomials are continuous everywhere, f is clearly continuous if $x \notin \{0,2\}$. For continuity at $x=0$, we require that the one-sided limits of $f(x)$ at $x=0$ be equal. But $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-}(ax+b) = b$ and $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+}(x^2+3a-b) = 3a-b$.
Similarly, for continuity at $x=2$, we require that the one-sided limits of $f(x)$ at $x=2$ be equal. But $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-}(x^2+3a-b) = 4+3a-b$ and $\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+}(3x-5) = 6-5 = 1$.
Equality of one-sided limits is equivalent to

$$b = 3a-b \text{ and } 4+3a-b = 1$$
$$\Rightarrow a = -2 \text{ and } b = -3.$$

p.83, pr.40

(b) $\boxed{\text{10 Points}}$ Define $g(3)$ that *extends* $g(x) = \dfrac{x^2-9}{x-3}$ to be *continuous* at $x=3$.

Solution:

$$\lim_{x\to 3} g(x) = \lim_{x\to 3}\frac{x^2-9}{x-3} = \lim_{x\to 3}\frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} = \lim_{x\to 3}\frac{x+3}{1} = 3+3 = 6$$

Hence we define $g(3) = 6$ and get the unique extension $\tilde{g}$.

$$\tilde{g}(x) = \begin{cases} \dfrac{x^2-9}{x-3}, & x \ne 3 \\ 6, & x = 3 \end{cases}$$

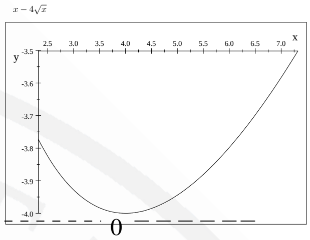
p.695, pr.37

3. (a) $\boxed{\text{13 Points}}$ Find all the extreme values (absolute and local) of $f(x) = x - 4\sqrt{x}$.

Solution: First the domain of f is $[0, +\infty)$. Note that

$$y' = 1 - \frac{4}{2\sqrt{x}} = \frac{\sqrt{x}-2}{\sqrt{x}},$$

which is zero at $x = 4$ and is undefined when $x = 0$. If $x \in (0,4)$, we have $y' < 0$ and if $x \in (4, +\infty)$, we have $y' > 0$. Hence there is a *local minimum* at $x = 4$. Now for the absolute extrema, we compare the values of f at 0 and 4 respectively. Hence $f(0) = 0 - 4\sqrt{0} = 0$ is the *local maximum* and $f(4) = 4 - 4\sqrt{4} = \boxed{-4}$ is the *absolute minimum*.



sign of y': − − − − − − 0 + + + + + + + + + + + + + + − − − − − − − 0 − − − − − −

0 0 4 $+\infty$ x

graph of y: concave down concave up concave down

p.879, pr.42

(b) $\boxed{\text{14 Points}}$ Use the formula $f'(x) = \lim_{z \to x} \frac{f(z)-f(x)}{z-x}$ to find the *derivative* of $g(x) = \frac{1}{x+2}$.

Solution: Here $f(z) = g(z) = \frac{1}{z+2}$ and $f(x) = g(x) = \frac{1}{x+2}$ and so

$$f'(x) = \lim_{z\to x} \frac{f(z)-f(x)}{z-x} = \lim_{z\to x} \frac{g(z)-g(x)}{z-x} = \lim_{z\to x} \frac{\frac{1}{z+2} - \frac{1}{x+2}}{z-x} = \lim_{z\to x} \frac{\frac{x+2-(z+2)}{(x+2)(z+2)}}{z-x}$$

$$= \lim_{z\to x} \frac{x-z}{(z-x)(x+2)(z+2)} = \lim_{z\to x} \frac{-\cancel{(z-x)}}{\cancel{(z-x)}(x+2)(z+2)} = \frac{-1}{(x+2)(x+2)} = \boxed{-\frac{1}{(x+2)^2}}$$

p.112, pr.26

4. (a) $\boxed{\text{15 Points}}$ Find the equation for the line that is **(a)** *tangent*, **(b)** *normal* to the curve $6x^2 + 3xy + 2y^2 + 17y - 6 = 0$ at the point $P_0(-1, 0)$.

Solution: Using implicit differenttiation, we have

$$6x^2 + 3xy + 2y^2 + 17y - 6 = 0 \Rightarrow 12x + 3y + 3xy' + 4yy' + 17y' = 0$$
$$\Rightarrow y'(3x + 4y + 17) = -12x - 3y$$
$$\Rightarrow y' = \frac{-12x - 3y}{3x + 4y + 17}$$
$$\Rightarrow \text{slope of tangent} = \left.\frac{-12x-3y}{3x+4y+17}\right|_{x=-1} = \frac{(-12)(-1)-(3)(0)}{(3)(-1)+(4)(0)+17} = \frac{12}{14} = \frac{6}{7}$$

(a) An equation for the tangent line is $y - 0 = \frac{6}{7}(x+1) \Rightarrow \boxed{6x - 7y = -6}$

(b) An equation for the normal line is $y - 0 = -\frac{7}{6}(x+1) \Rightarrow \boxed{7x + 6y = -7}$

p.695, pr.37

(b) 11 Points Find dy/dx if $y = \left(3+\cos^3(3x)\right)^{-1/3}$.

Solution:

$$\begin{aligned}
\frac{dy}{dx} = \frac{d}{dx}\left[\left(3+\cos^3(3x)\right)^{-1/3}\right] &= -\frac{1}{3}\left(3+\cos^3(3x)\right)^{-4/3}\frac{d}{dx}\left(3+\cos^3(3x)\right)\\
&= -\frac{1}{3}\left(3+\cos^3(3x)\right)^{-4/3}\left(0+3\cos^2(3x)\frac{d}{dx}(\cos(3x))\right)\\
&= -\frac{1}{3}\left(3+\cos^3(3x)\right)^{-4/3}3\cos^2(3x)(-3\sin(3x))\\
&= \boxed{3\left(3+\cos^3(3x)\right)^{-4/3}\cos^2(3x)\sin(3x)}
\end{aligned}$$

p.452, pr.24