Your Name / Adınız - Soyadınız

Your Signature / İmza

Student ID # / Öğrenci No

Professor's Name / Öğretim Üyesi

Your Department / Bölüm

- Calculators, cell phones off and away!.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. **Show your work in evaluating any limits, derivatives**.
- Place a box around your answer to each question.
- Use a **BLUE ball-point pen** to fill the cover sheet. Please make sure that your exam is complete.
- Time limit is 70 min.

Do not write in the table to the right.

Problem	Points	Score
1	18	
2	22	
3	24	
4	16	
5	20	
Total:	100	

1. Evaluate the following integrals.

(a) 10 Points $\displaystyle\int_1^4 \frac{10\sqrt{v}}{(1+v^{3/2})^2}\,dv.$

Solution: Let $u = 1+v^{3/2}$. Then $du = \frac{3}{2}v^{1/2}\,dv$. When $v=1$, we have $u=2$ and when $v=4$, we have $u=9$. Therefore

$$\begin{aligned}\int_1^4 \frac{10\sqrt{v}}{(1+v^{3/2})^2}\,dv &= (10)(\frac{2}{3})\int_1^4 \frac{1}{(1+v^{3/2})^2}\frac{3}{2}v^{1/2}\,dv\\ &= \frac{20}{3}\int_2^9 \frac{1}{u^2}\,dv\\ &= \frac{20}{3}\left[-\frac{1}{u}\right]_2^9 = \frac{20}{3}\left[-\frac{1}{9}+\frac{1}{2}\right] = \boxed{\frac{70}{27}}\end{aligned}$$

p.583, pr.32

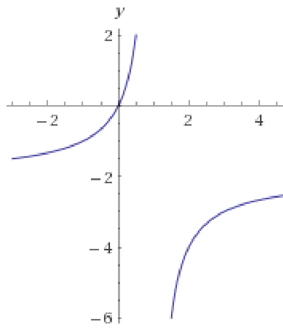
(b) 8 Points $\displaystyle\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\,dt.$

Solution: Let $u=\cos(2t+1)$. Then $du = -2\sin(2t+1)\,dt$. Now we have

$$\begin{aligned}\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\,dt &= -\frac{1}{2}\int \frac{1}{\cos^2(2t+1)}(-2)\sin(2t+1)\,dt\\ &= -\frac{1}{2}\int \frac{1}{u^2}\,du\\ &= -\frac{1}{2}\left[-\frac{1}{u}\right]+C = \frac{1}{2\cos(2t+1)}+C = \boxed{\frac{1}{2}\sec(2t+1)+C}\end{aligned}$$

p.452, pr.24

2. (a) $\boxed{\text{12 Points}}$ Find the normals to the curve $xy+2x-y=0$ that are parallel to the line $2x+y=0$.



Solution: If a normal to a curve is parallel to $2x+y=0$ then these two lines have the same slope:

$y=-2x$

slope $=-2$

Since the normal line at a point is perpendicular to the tangent line at that point, the slope of the tangent line is $1/2$

$xy+2x-y=0 \Rightarrow$ implicit differentiation

$xy'+y+2-y'=0$

$y'(x-1)=-y-2$

$y'=(-y-2)/(x-1) \Rightarrow$ equate to slope of the tangent $(1/2)$

$1/2=(-y-2)/(x-1)$

$y=(-1/2)x-3/2 \Rightarrow$the equation of the tangent, solve as a system with the curve to get the intersections:

$xy+2x-y=0$

$x=-1,3$

$y=-1\,,-3$

$m=-2\,,(-1,-1)$

$y-y_1=m(x-x_1)$

$y+1=-2(x+1)$

$y=-2x-3 \Rightarrow$ equation of the normal at $(-1,-1)$

$y+3=-2(x-3)$

$y=-2x+3 \Rightarrow$ equation of the normal at $(3,-3)$

p.487, pr.6

(b) $\boxed{\text{10 Points}}$ Show that the function $f(x)=x^4+3x+1$ has *exactly one zero* in $[-2,-1]$.

Solution: The function $f(x)=x^4+3x+1$ is decreasing on the interval $[-2,-1]$. To show this, we take the derivative.

$f'(x)=4x^3+3$, which is less than zero whenever

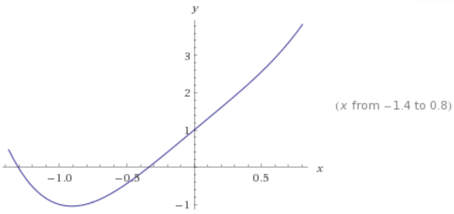
$4x^3<-3$, which is the same as $x^3<-3/4$, or equivalently

$x<(-3/4)^{(1/3)}$. Since $(-3/4)^{(1/3)}$ is greater than -1, the derivative is negative on the whole interval, which means the function is decreasing and thus can have at most one zero there.

Now, $f(-2)=(-2)^4+3(-2)+1=11$, which is greater than 0, and $f(-1)=(-1)^4+3(-1)+1=-1$, less than 0.

Combining all of this, we can say that the graph of $f(x)=x^4+3x+1$ crosses the x- axis exactly once on the interval $[-2,-1]$, and so has exactly one zero there.

Here's the graph to illustrate the solution.



p.413, pr.58

3. Given the curve $y = \dfrac{4x}{x^2+4}$ and derivatives $y' = \dfrac{-4x^2+16}{(x^2+4)^2}$ and $y'' = \dfrac{8x^3-96x}{(x^2+4)^3}$

(a) $\boxed{\text{3 Points}}$ Identify the *domain* of f and any *symmetries* the curve may have.

Solution: Domain is $(-\infty,+\infty)$. Since $y(-x) = \dfrac{4(-x)}{(-x)^2+4} = -\dfrac{4x}{x^2+4} = -y(x)$ for each $x \in (-\infty,+\infty)$, the function is odd and so graph is symmetric with respect to the origin.

p.212, pr.92

(b) $\boxed{\text{5 Points}}$ Find the intervals where the graph is increasing and decreasing. Find the local maximum and minimum values.

Solution:
$y' = \dfrac{-4x^2+16}{(x^2+4)^2} > 0$ if and only if $-4x^2+16>0$, that is iff $x^2<4$, i.e., graph is increasing on $(-2,+2)$ $-2<x<2$ and decreasing on $(-\infty,-2)\cup(2,+\infty)$.. By the First Derivative Test, graph has a local minimum at $x=-2$ and a local maximum at $x=2$. The local minimum value is $f(-2)=-1$ which is the absolute mi. value and local maximum value is $f(2)=1$ which is the absolute max. value.

p.241, pr.45

(c) $\boxed{\text{5 Points}}$ Determine where the graph is concave up and concave down, and find any inflection points.

Solution:
Notice that $y'' = \dfrac{8x^3-96x}{(x^2+4)^3} > 0$ if and only if $8x^3-96x>0$, that is iff $x(x^2-12)>0$. solving the last inequality, we see that graph is concave up on $(-2\sqrt{3},0)\cup(2\sqrt{3},+\infty)$ and concave down on $(-\infty,-2\sqrt{3})\cup(0,2\sqrt{3})$. Now due to the sign changes in y'', there are three points of inflection namely, $(-2\sqrt{3},-\sqrt{3}/2)$, $(0,0)$, and $(2\sqrt{3},\sqrt{3}/2))$.

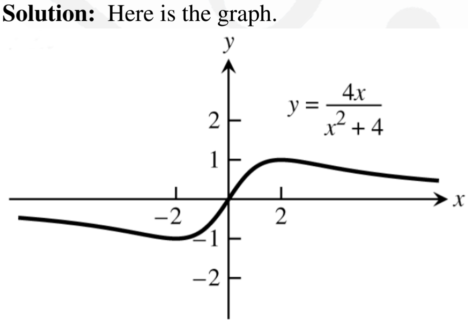
p.241, pr.45

(d) $\boxed{\text{5 Points}}$ Find the asymptotes.

Solution: Since $x^2+4\neq 0$, for each $x\in(-\infty,\infty)$, graph can not have a vertical asymptote. Since $\lim\limits_{x\to\pm\infty}\dfrac{4x}{x^2+4}=0$, we see that $y=0$ is a horizontal asymptote. p.241, pr.45

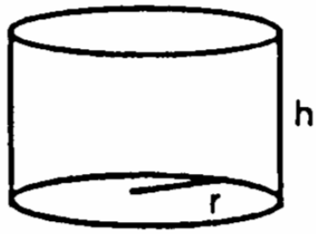
(e) $\boxed{\text{6 Points}}$ Draw the graph of f on the given grid showing all significant features.

Solution: Here is the graph.



p.241, pr.45

4. $\boxed{\text{16 Points}}$ What are the dimensions of the *lightest* (with minimum area) open-top right circular cylindrical can that will hold a volume of $1000cm^3$?



Solution: Surface area of a cylinder $= 2\pi rh$

Surface area with one end cap $= 2\pi rh + \pi r^2$

$S = 2\pi rh + \pi r^2$

$V = \pi r^2 h = 1000cm^3$

$h = 1000/\pi r^2$

Substitute in for h...

$S = 2\pi r(1000/\pi r^2) + \pi r^2$

$S = (2000/r) + \pi r^2$

Take the derivative and set it equal to zero to find the minimum...

$dS/dr = (-2000/r^2) + 2\pi r$

$(-2000/r^2) + 2\pi r = 0$

$-2000 + 2\pi r^3 = 0$

$r^3 = 2000/2\pi$

$r = \sqrt[3]{1000/\pi}$

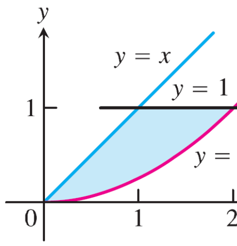
$\left[\frac{d^2S}{dr^2}\right]_{r=\sqrt[3]{1000/\pi}} = (4000/r^3) + 2\pi = 4\pi + 2\pi = 6\pi$ which is positive and Second Derivative Test yields a minimum.

$r = 6.83cm$

$h = 1000/\pi(6.83)^2 = 6.82cm$

p.695, pr.37

5. $\boxed{\text{20 Points}}$ Find the total area of the shaded region.



Solution: Let R denote the shaded region. We shall find the area of R in two different ways. One description of R is

$$R = \underbrace{\{(x,y) \in \mathbb{R}^2 | \quad \frac{x^2}{4} \leq y \leq x, \quad 0 \leq x \leq 1\}}_{R_1} \cup \underbrace{\{(x,y) \in \mathbb{R}^2 | \quad \frac{x^2}{4} \leq y \leq 1, \quad 1 \leq x \leq 2\}}_{R_2}$$

Notice that $R = R_1 \cup R_2$ and $R_1 \cap R_2 = \emptyset$. Therefore $A(R) = A(R_1) + A(R_2)$. This shows that we will need two integrals.

$$\begin{aligned} A(R) = A(R_1) + A(R_2) &= \int_0^1 \left(x - \frac{x^2}{4}\right) \mathrm{d}x + \int_1^2 \left(1 - \frac{x^2}{4}\right) \mathrm{d}x \\ &= \left[\frac{1}{2}x^2 - \frac{x^3}{12}\right]_0^1 + \left[x - \frac{x^3}{12}\right]_1^2 \\ &= 2 - \frac{8}{12} - \frac{1}{2} = \boxed{\frac{5}{6}} \end{aligned}$$

Next, we will find the area by integrating with respect to y. Another description for R is

$$R = \{(x,y) \in \mathbb{R}^2 | \quad y \leq x \leq 2\sqrt{y}, \quad 0 \leq y \leq 1\}$$

This will, of course, require a single integral (easier than the first way). Hence

$$\begin{aligned} A = A(R) &= \int_0^1 (2\sqrt{y} - y)\, dy \\ &= \left[2\frac{y^{3/2}}{3/2} - \frac{1}{2}y^2\right]_0^1 = \frac{4}{3} - \frac{1}{2} = \boxed{\frac{5}{6}} \end{aligned}$$

p.298, pr.36