OKAN ÜNİVERSİTESİ
MÜHENDİSLİK FAKÜLTESİ
MÜHENDİSLİK TEMEL BİLİMLERİ BÖLÜMÜ

2018.05.14, 13:10–14:40 | **MATH113 Mathematics I – Final Exam** | N. Course

FORENAME:

SURNAME:

STUDENT NO:

SIGNATURE:

Time Allowed: **90** min.

You must answer all questions.

Do not open the exam until you are told that you may begin.
Sınavın başladığı yüksek sesle söylenene kadar sayfayı çevirmeyin.

1. You will have **90** minutes.
2. The points awarded for each part, of each question, are stated next to it.
3. All of the questions are in English. You must answer in English.
4. You must show your working for all questions.
5. This exam contains 8 pages. Check to see if any pages are missing.
6. If you wish to leave before the end of the exam, give your exam script to an invigilator and leave the room quietly. You may not leave in the first 20 minutes, or in the final 10 minutes, of the exam.
7. Switch your mobile phone off and seal it in the envelope provided. Do not open your envelope until the exam is finished or you have left the room.
8. All communication between students, either verbally or non-verbally, is strictly forbidden. Students who finish early must leave the room without communicating with other students.
9. Calculators, mobile phones and any digital means of communication are forbidden. The sharing of pens, erasers or any other item between students is forbidden.
10. All bags, coats, books, notes, etc. must be placed away from your desks and away from the seats next to you. You may not access these during the exam. Take out everything that you will need before the exam starts.
11. Any student found cheating or attempting to cheat will receive a mark of zero (0), and will be investigated according to the regulations of Yükseköğretim Kurumları Öğrenci Disiplin Yönetmeliği.

1. Sınav süresi toplam **90** dakikadır.
2. Soruların her bölümünün kaç puan olduğu yanlarında belirtilmiştir.
3. Tüm sorular İngilizce'dir. Cevaplarınızı İngilizce veriniz.
4. Sonuca ulaşmak için yaptığınız işlemleri ayrıntılarıyla gösteriniz.
5. Sınav 8 sayfadan oluşmaktadır. Lütfen eksik sayfa olup olmadığını kontrol edin.
6. Sınav süresi sona ermeden sınavınızı teslim edip çıkmak isterseniz, sınav kağıdınızı gözetmenlerden birine veriniz ve sınav salonundan sessizce çıkınız. Sınavın ilk 20 dakikası ve son 10 dakikası içinde sınav salonundan çıkmanız yasaktır.
7. Cep telefonunuzu kapatınız ve size verilen zarfın içine koyunuz. Zarfı, sınav süresi bitene kadar ya da sınav salonundan çıkana kadar açmayınız.
8. Sınav esnasında öğrenciler arasında, sözlü ya da sözsüz, her türlü iletişim kesinlikle yasaktır. Sınavını erken bitiren öğrenciler, diğer öğrencilerle hiç bir şekilde iletişim kurmadan sessizce sınıftan çıkmalıdır.
9. Sınav esnasında hesap makinesi, cep telefonu ve dijital bilgi alışverişi yapılan her türlü malzemelerin kullanımı ile diğer silgi, kalem, vb. alışverişlerin yapılması kesinlikle yasaktır.
10. Çanta, palto, kitap ve ders notlarınız gibi eşyalarınız sıraların üzerinden ve yanınızdaki sandalyeden kaldırılmalıdır. Sınav süresince bu tür eşyaları kullanmanız yasaktır, bu nedenle ihtiyacınız olacak herşeyi sınav başlamadan yanınıza alınız.
11. Her türlü sınav, ve diğer çalışmada, kopya çeken veya kopya çekme girişiminde bulunan bir öğrenci, o sınav ya da çalışmadan sıfır (0) not almış sayılır, ve o öğrenci hakkında Yükseköğretim Kurumları Öğrenci Disiplin Yönetmeliği hükümleri uyarınca disiplin kovuşturması yapılır.

α

1	2	3	4	5	6	TOTAL
/20	/1	/20	/20	/20	/19	/100

Definition *We write* $\lim_{x \to c} f(x) = L$ *if and only if, for all* $\varepsilon > 0$*, there exists* $\delta > 0$ *such that for all* x*,*

$$0 < |x - c| < \delta \qquad \Longrightarrow \qquad |f(x) - L| < \varepsilon.$$

Question 1 (The Precise Definition of a Limit)

[20 pts] Use the definition above to prove that $\lim_{x \to 2} \dfrac{3x^2 - 12}{x - 2} = 12$.

Let $\varepsilon > 0$. Choose $\delta = \frac{\varepsilon}{3}$. Then for all x we have that

$$\begin{aligned} 0 < |x - 2| < \delta \qquad \Longrightarrow \qquad \left| \frac{3x^2 - 12}{x - 2} - 12 \right| &= \left| \frac{3(x-2)(x+2)}{x - 2} - 12 \right| \\ &= 3\,|(x+2) - 4| \\ &= 3\,|x - 2| \\ &< 3\delta \\ &= \varepsilon. \end{aligned}$$

Therefore $\lim_{x \to 2} \frac{3x^2 - 12}{x - 2} = 12$.

Question 2 [1 pt] Please write your student number at the top-right of this page.

Question 3 (Arc Length)

[20 pts] Find the length of the curve $y = \frac{x^4}{4} + \frac{1}{8x^2}$ from $x = 1$ to $x = 2$.

We calculate that

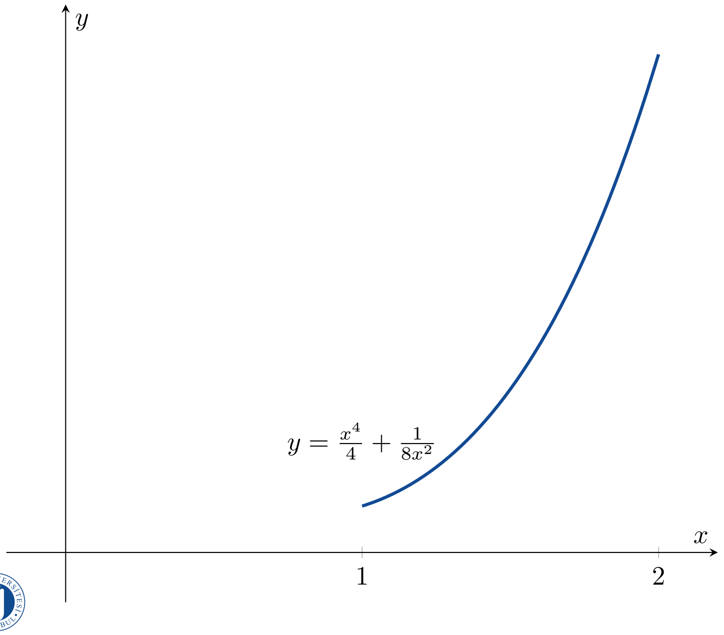
$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^4}{4} + \frac{1}{8x^2}\right) = x^3 - \frac{2}{8x^3} = x^3 - \frac{1}{4x^3}$$

and

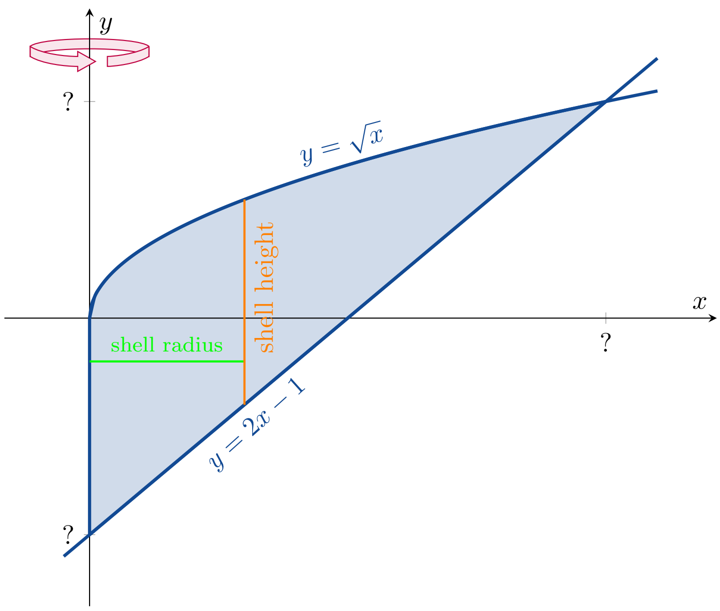
$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + x^6 - \frac{1}{2} + \frac{1}{16x^6} = x^6 + \frac{1}{2} + \frac{1}{16x^6} = \left(x^3 + \frac{1}{4x^3}\right)^2.$$

The length of the curve is

$$L = \int_1^2 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_1^2 x^3 + \frac{1}{4x^3}\, dx = \left[\frac{x^4}{4} - \frac{1}{8x^2}\right]_1^2 = \left(\frac{16}{4} - \frac{1}{32}\right) - \left(\frac{1}{4} - \frac{1}{8}\right)$$
$$= \frac{128}{32} - \frac{1}{32} - \frac{8}{32} + \frac{4}{32} = \frac{123}{32}.$$



Length of curve =



Question 4 (Volumes Using Cylindrical Shells) The region bounded by $y=\sqrt{x}$, $y=2x-1$ and $x=0$ is shown above. This region is rotated about the y-axis to generate a solid.

[20 pts] **<u>Use the shell method</u>** to calculate the volume of the solid.

First we calculate that

$$\begin{aligned}2x-1&=\sqrt{x}\\4x^2-4x+1&=x\\4x^2-5x+1&=0\\x^2-\frac{5}{4}x+\frac{1}{4}&=0\\(x-1)(x-\frac{1}{4})&=0.\end{aligned}$$

Since $2\left(\frac{1}{4}\right)-1=-\frac{1}{2}\neq\frac{1}{2}=\sqrt{\frac{1}{4}}$, the curves must intersect when $x=1$.
As shown on the figure above, we have that

$$\begin{aligned}\text{shell radius}&=x\\\text{shell height}&=\sqrt{x}-(2x-1)=\sqrt{x}-2x+1.\end{aligned}$$

Therefore the volume of the solid is

$$\begin{aligned}V&=\int_0^1 2\pi\binom{\text{shell}}{\text{radius}}\binom{\text{shell}}{\text{height}}\,dx\\&=2\pi\int_0^1 x^{\frac{3}{2}}-2x^2+x\,dx\\&=2\pi\left[\frac{2}{5}x^{\frac{5}{2}}-\frac{2}{3}x^3+\frac{1}{2}x^2\right]_0^1\\&=2\pi\left(\frac{2}{5}-\frac{2}{3}+\frac{1}{2}\right)-2\pi(0-0+0)\\&=\frac{7\pi}{15}.\end{aligned}$$

Volume =

Question 5 (Inverse Trigonometric Functions)

[20 pts] Calculate $\displaystyle\int \frac{3\ dr}{\sqrt{5-4r^2+8r}}$.

[HINT: First write $5-4r^2+8r$ in the form a^2-u^2.]

First note that

$$5-4r^2+8r = 5-4(r^2-2r) = 5-4(r^2-2r+1-1) = 5-4(r^2-2r+1)+4 = 9-4(r-1)^2$$

Let $u = 2(r-1) = 2r-2$. Then $du = \frac{du}{dr}dr = 2\ dr$. It follows that

$$\int \frac{3\ dr}{\sqrt{5-4r^2+8r}} = \int \frac{3\ dr}{\sqrt{9-4(r-1)^2}} = \frac{3}{2}\int \frac{du}{\sqrt{3^2-u^2}} = \frac{3}{2}\sin^{-1}\frac{u}{3} + C = \frac{3}{2}\sin^{-1}\left(\frac{2r-2}{3}\right) + C.$$

$$\int \frac{3\ dr}{\sqrt{5-4r^2+8r}} = \boxed{}$$

Question 6 (Indeterminate Forms)

[19 pts] Calculate $\lim_{x \to \frac{\pi}{2}^-} \left(x - \frac{\pi}{2}\right) \sec x$.

[State which rules, if any, you are using.]

We calculate that

$$\begin{aligned}
\lim_{x \to \frac{\pi}{2}^-} \left(x - \frac{\pi}{2}\right) \sec x &= \lim_{x \to \frac{\pi}{2}^-} \left(x - \frac{\pi}{2}\right) \left(\frac{1}{\cos x}\right) \\
&= \lim_{x \to \frac{\pi}{2}^-} \frac{x - \frac{\pi}{2}}{\cos x} \\
&= \lim_{x \to \frac{\pi}{2}^-} \frac{\frac{d}{dx}\left(x - \frac{\pi}{2}\right)}{\frac{d}{dx}\cos x} \\
&= \lim_{x \to \frac{\pi}{2}^-} \frac{1}{-\sin x} \\
&= \frac{1}{-1} \\
&= -1
\end{aligned}$$

by l'Hôpital's Rule.

$$\lim_{x \to \frac{\pi}{2}^-} \left(x - \frac{\pi}{2}\right) \sec x = \boxed{}$$

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$$\cos\theta = \sin\left(\tfrac{\pi}{2} - \theta\right)$$
$$\cos^2\theta + \sin^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos^2\theta = \tfrac{1}{2}(1+\cos 2\theta)$$
$$\sin^2\theta = \tfrac{1}{2}(1-\cos 2\theta)$$

$\cos 0 = \cos 0^\circ = 1$	$\sin 0 = \sin 0^\circ = 0$
$\cos\frac{\pi}{6} = \cos 30^\circ = \frac{\sqrt{3}}{2}$	$\sin\frac{\pi}{6} = \sin 30^\circ = \frac{1}{2}$
$\cos\frac{\pi}{4} = \cos 45^\circ = \frac{1}{\sqrt{2}}$	$\sin\frac{\pi}{4} = \sin 45^\circ = \frac{1}{\sqrt{2}}$
$\cos\frac{\pi}{3} = \cos 60^\circ = \frac{1}{2}$	$\sin\frac{\pi}{3} = \sin 60^\circ = \frac{\sqrt{3}}{2}$
$\cos\frac{\pi}{2} = \cos 90^\circ = 0$	$\sin\frac{\pi}{2} = \sin 90^\circ = 1$

$$(uv)' = uv' + u'v$$
$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$
$$(f\circ g)'(x) = f'(g(x))\,g'(x)$$
$$\left(f^{-1}\right)'(x) = \frac{1}{f'\left(f^{-1}(x)\right)}$$

$$\operatorname{av}(f) = \frac{1}{b-a}\int_a^b f(x)\,dx$$
$$V = \int_a^b A(x)\,dx$$
$$V = \int_a^b \pi\big(R(x)\big)^2\,dx$$
$$V = \int_a^b \pi\Big(\big(R(x)\big)^2 - \big(r(x)\big)^2\Big)\,dx$$
$$V = \int_a^b 2\pi\binom{\text{shell}}{\text{radius}}\binom{\text{shell}}{\text{height}}\,dx$$
$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$
$$S = \int_a^b 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

$$\frac{d}{dx}x^n = nx^{n-1}$$
$$\frac{d}{dx}\sin x = \cos x$$
$$\frac{d}{dx}\cos x = -\sin x$$

$\tan x = \frac{\sin x}{\cos x}$ $\qquad \frac{d}{dx}\tan x = \sec^2 x$

$$\int \tan x\,dx = \ln|\sec x| + C$$

$\sec x = \frac{1}{\cos x}$ $\qquad \frac{d}{dx}\sec x = \sec x\tan x$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$\cot x = \frac{\cos x}{\sin x}$ $\qquad \frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$

$$\int \cot x\,dx = \ln|\sin x| + C$$

$\operatorname{cosec} x = \csc x = \frac{1}{\sin x}$ $\qquad \frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x\cot x$

$$\int \operatorname{cosec} x\,dx = -\ln|\operatorname{cosec} x + \cot x| + C$$

$$\frac{d}{dx}\sin^{-1}\frac{x}{a} = \frac{1}{\sqrt{a^2-x^2}}$$
$$\frac{d}{dx}\tan^{-1}\frac{x}{a} = \frac{a}{a^2+x^2}$$
$$\frac{d}{dx}\sec^{-1}\frac{x}{a} = \frac{a}{|x|\sqrt{x^2-a^2}}$$
$$\frac{d}{dx}e^x = e^x$$
$$\frac{d}{dx}\ln|x| = \frac{1}{x}$$

$\sinh x = \frac{e^x - e^{-x}}{2}$	$\cosh x = \frac{e^x + e^{-x}}{2}$
$\cosh^2 x - \sinh^2 x = 1$	
$\sinh 2x = 2\sinh x\cosh x$	$\cosh 2x = \cosh^2 x - \sinh^2 x$
$\cosh^2 x = \frac{\cosh 2x + 1}{2}$	$\sinh^2 x = \frac{\cosh 2x - 1}{2}$
$\tanh^2 x = 1 - \operatorname{sech}^2 x$	$\coth^2 x = 1 + \operatorname{cosech}^2 x$
$\frac{d}{dx}\sinh x = \cosh x$	$\frac{d}{dx}\cosh x = \sinh x$
$\frac{d}{dx}\tanh x = \operatorname{sech}^2 x$	$\frac{d}{dx}\coth x = -\operatorname{cosech}^2 x$
$\frac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x\tanh x$	$\frac{d}{dx}\operatorname{cosech} x = -\operatorname{cosech} x\coth x$

$\operatorname{sech}^{-1} x = \cosh^{-1}\frac{1}{x}$ $\qquad \operatorname{cosech}^{-1} x = \sinh^{-1}\frac{1}{x}$ $\qquad \coth^{-1} x = \tanh^{-1}\frac{1}{x}$

$\frac{d}{dx}\sinh^{-1}\frac{x}{a} = \frac{1}{\sqrt{a^2+x^2}}$	$(a > 0)$
$\frac{d}{dx}\cosh^{-1}\frac{x}{a} = \frac{1}{\sqrt{x^2-a^2}}$	$(x > a > 0)$
$\frac{d}{dx}\tanh^{-1}\frac{x}{a} = \frac{a}{a^2-x^2}$	$(x^2 < a^2)$
$\frac{d}{dx}\coth^{-1}\frac{x}{a} = \frac{a}{a^2-x^2}$	$(x^2 > a^2)$
$\frac{d}{dx}\operatorname{sech}^{-1}\frac{x}{a} = \frac{-a}{x\sqrt{a^2-x^2}}$	$(0 < x < 1)$
$\frac{d}{dx}\operatorname{cosech}^{-1}\frac{x}{a} = \frac{-a}{|x|\sqrt{a^2+x^2}}$	$(x \neq 0,\ a > 0)$