FORENAME:

SURNAME:

STUDENT NO:

DEPARTMENT:

TEACHER: ☐ Neil Course ☐ Vasfi Eldem ☐ Asuman Özer ☐ Sezgin Sezer

SIGNATURE:

Question	Points	Score
1	20	
2	30	
3	30	
4	20	
Total:	100	

- The time limit is 90 minutes.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$), except as noted in particular problems.
- All communication between students, either verbally or non-verbally, is strictly forbidden.
- Calculators, mobile phones, smart watches, and any digital means of communication are forbidden. The sharing of pens, erasers or any other item between students is forbidden.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- Please do not write in the table above.

1. (a) $\boxed{\text{10 points}}$ Suppose that $\lim\limits_{x\to\sqrt{5}} \dfrac{1}{x+g(x)} = 2$. Find $\lim\limits_{x\to\sqrt{5}} g(x)$.

Solution:

$$\lim_{x\to\sqrt{5}} \frac{1}{x+g(x)} = \frac{\lim\limits_{x\to\sqrt{5}} 1}{\lim\limits_{x\to\sqrt{5}} x + \lim\limits_{x\to\sqrt{5}} g(x)} = 2$$

$$\Rightarrow \frac{1}{\sqrt{5} + \lim\limits_{x\to\sqrt{5}} g(x)} = 2 \Rightarrow = \frac{1}{2} - \sqrt{5}$$

(b) $\boxed{\text{10 points}}$ Find a positive number $x \in (0,\infty)$ for which the number $\left(x+\frac{1}{x}\right)$ is the smallest (least) possible.

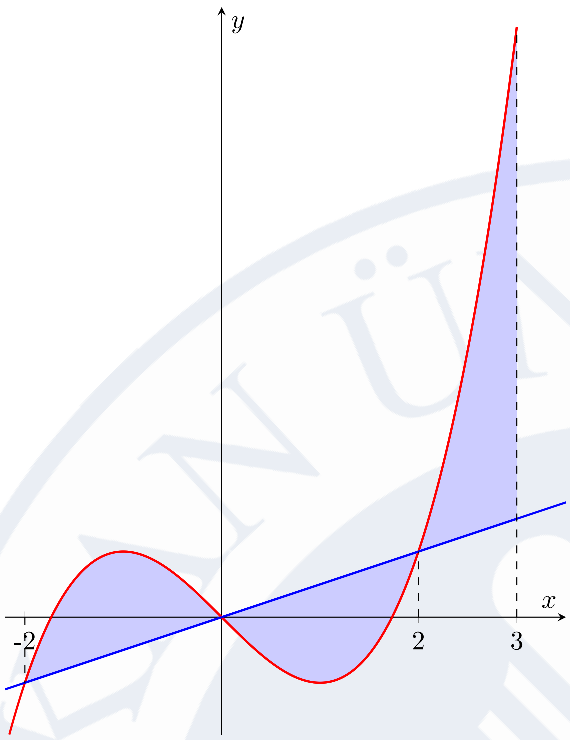
Solution: Let this number be x. So, its reciprocal will be $\frac{1}{x}$.

$$f(x) = x + \frac{1}{x} = \frac{x^2-1}{x^2}$$

$$f'(x) = \frac{x^2-1}{x} = \frac{(x-1)(x+1)}{x^2} = 0$$

Critical points: $x=-1$, $x=0$, $x=1$. The number is $x=1$, because the number is positive. In addition, the sum is $f(1)=2$.

2. (a) 15 points Find the area of the region enclosed by the curve $y = \frac{x^3}{3} - x$ and the line $y = \frac{x}{3}$ for $-2 \leq x \leq 3$.



Solution:

$$A = \left|\int_{-2}^{0} [(\frac{x^3}{3} - x) - \frac{x}{3}]dx\right| + \left|\int_{0}^{2} [\frac{x}{3} - (\frac{x^3}{3} - x)]dx\right| + \left|\int_{2}^{3} [(\frac{x^3}{3} - x) - x]dx\right|$$

$$= \left|\int_{-2}^{0} [\frac{x^3}{3} - \frac{4x}{3}]dx\right| + \left|\int_{0}^{2} [\frac{4x}{3} - \frac{x^3}{3}]dx\right| + \left|\int_{2}^{3} [\frac{x^3}{3} - \frac{4x}{3}]dx\right|$$

$$= \left|\frac{x^4}{12} - \frac{2x^2}{3}\right|_{-2}^{0} + \left|\frac{2x^2}{3} - \frac{x^4}{12}\right|_{0}^{2} + \left|\frac{x^4}{12} - \frac{2x^2}{3}\right|_{2}^{3} = |-\frac{4}{3}| + |\frac{4}{3}| + |\frac{51}{3}| = \frac{59}{3}$$

(b) 15 points Find the surface area of the surface generated by revolving the curve $x = 2\sqrt{4-y}$, $0 \leq y \leq \frac{15}{4}$ about the y-axis.

Solution:

$$\frac{dx}{dy} = \frac{-1}{\sqrt{4-y}} \Rightarrow A = \int_{0}^{\frac{15}{4}} 2\pi x \sqrt{1 + (\frac{dx}{dy})^2} dy = \int_{0}^{\frac{15}{4}} 2\pi(2\sqrt{4-y})\sqrt{1 + \frac{1}{4-y}} dy$$

$$= 4\pi \int_{0}^{\frac{15}{4}} \sqrt{4-y} \frac{5-y}{\sqrt{4-y}} dy = 4\pi \int_{0}^{\frac{15}{4}} (5-y)dy = 4\pi(5y - \frac{y^2}{2})\Big|_{0}^{\frac{15}{4}} = 15\frac{25}{8}\pi = \frac{375}{8}\pi$$

3. $\boxed{\text{30 points}}$ Compute the volume of the solid generated by revolving the region bounded by $y = x$ and $y = x^2$ **a**bout y-axis using

(i). the shell method

(ii). the washer method.

Solution:

(i). **the shell method** Shell radius: x
Shell high: $x - x^2$

$$V = \int_0^1 2\pi x(x - x^2)dx = 2\pi \int_0^1 (x^2 - x^3)dx$$
$$= 2\pi\left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1 = 2\pi\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{\pi}{6}$$

(ii). **the washer method** Outer radius: $R(y) = \sqrt{y}$
İnner radius: $r(y) = y$

$$V = \int_0^1 \pi[(R(y))^2 - (r(y))^2]dy = \pi \int_0^1 [(\sqrt{y})^2 - y^2]dy$$
$$= \pi \int_0^1 [y - y^2]dy = \pi\left(\frac{y^2}{2} - \frac{y^3}{3}\right) = \frac{\pi}{6}$$

4. $\boxed{\text{20 points}}$ Sketch the graph of the function $y = x^{2/3}(x-5)$: $y' = \dfrac{5(x-2)}{3x^{\frac{1}{3}}}$, $y'' = \dfrac{10}{9}\dfrac{x+1}{x^{\frac{4}{3}}}$.

Solution:

(i). Critical Points: $x = 0$ and $x = 2$.

(ii). Local max: $f(0) = 0$, Local min: $f(2) = -3\sqrt[3]{4}$, inf. point: $f(-1) = -6$

(iii). There is no asymptotes!

x	$-\infty$		-1		0		2		∞
$f'(x)$		+		+		-		+	
$f''(x)$		-		+		+		+	
$f(x)$