Your Name / Adınız - Soyadınız

Your Signature / İmza

Student ID # / Öğrenci No

Professor' s Name / Öğretim Üyesi

Your Department / Bölüm

- Calculators, cell phones off and away!.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. **Show your work in evaluating any limits, derivatives**.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- Use a **BLUE ball-point pen** to fill the cover sheet. Please make sure that your exam is complete.
- Time limit is 75 min.

Do not write in the table to the right.

Problem	Points	Score
1	15	
2	20	
3	30	
4	20	
5	35	
Total:	120	

1. $\boxed{\text{15 Points}}$ Let $f(x) = \sqrt{19-x}$, $L = 3$, $c = 10$, and $\varepsilon = 1$. Give a value for $\delta > 0$ such that for all x satisfying $0 < |x-c| < \delta$ the inequality $|f(x) - L| < \varepsilon$ holds.

Solution:

$$\begin{aligned}
|x-10| < \delta \Rightarrow & |f(x)-3| < \varepsilon \\
\Rightarrow & |f(x)-L| = |\sqrt{19-x}-3| < 1 \\
\Rightarrow & -1 < \sqrt{19-x}-3 < 1 \\
\Rightarrow & 2 < \sqrt{19-x} < 4 \\
\Rightarrow & 4 < 19-x < 16 \\
\Rightarrow & -16 < x-19 < -4 \\
\Rightarrow & -7 < x-10 < 5 \\
\Rightarrow & 5 < x-10 < 7
\end{aligned}$$

$\delta = min\{|-7|, |5|\} = 5$

2. Calculate the following limit. **(Answer any one of the following questions only. If you solve both, then first question will be evaluated and the other will be considered unanswered!)**

(a) $\boxed{\text{10 Points}}$ $\lim\limits_{x\to 0} \frac{\tan 2x}{x}$

Solution:

$$\begin{aligned}\lim_{x\to 0} \frac{\sin 2x}{x\cos 2x} &= 2\lim_{x\to 0} \frac{\sin 2x}{2x} \lim_{x\to 0} \cos 2x \\ &= 2\cdot 1\cdot 1 \\ &= 2\end{aligned}$$

(b) $\boxed{\text{10 Points}}$ $\lim\limits_{x\to\infty} \sqrt{x+9}-\sqrt{x+4}$

Solution:

$$\begin{aligned}&\lim_{x\to\infty} \frac{(\sqrt{x+9}-\sqrt{x+4})(\sqrt{x+9}+\sqrt{x+4})}{\sqrt{x+9}+\sqrt{x+4}} \\ &= \lim_{x\to\infty} \frac{5}{\sqrt{x+9}+\sqrt{x+4}} \\ &= 0\end{aligned}$$

3. (a) $\boxed{\text{15 Points}}$ Find the slope of the graph of $x+\sqrt{xy}=6$ at the point $(4,1)$. Then find equations for the lines that are tangent to the curve at the given point.

Solution: Slope:

$$\begin{aligned}1+\frac{y+xy'}{\sqrt{xy}} &= 0 \\ y' &= -\frac{\sqrt{xy}}{x} \\ y'|_{(4,1)} &= -\frac{1}{2}\end{aligned}$$

Equation of tangent line:

$$\begin{aligned}y-1 &= -\frac{1}{2}(x-4) \\ 2y-2 &= -x+4 \\ 2y+x-6 &= 0\end{aligned}$$

(b) $\boxed{\text{15 Points}}$ Find all values of the constants a and b for which the following function continuous and differentiable at $x = \pi$.

$$f(x) = \begin{cases} \sin x & x < \pi \\ ax + b & x \geq \pi. \end{cases}$$

Solution:

$$\lim_{x\to 0^-} \frac{f(\pi+h) - f(\pi)}{h} = \lim_{x\to 0^-} \frac{\sin(\pi+h) - \sin\pi}{h} \lim_{x\to 0^-} \frac{\sin\pi\cos h + \cos\pi\sin h}{h} = \lim_{x\to 0^-} (\frac{-\sin h}{h}) = -1$$

$$\lim_{x\to 0^+} \frac{f(\pi+h) - f(\pi)}{h} = \lim_{x\to 0^+} \frac{a(\pi+h) + b - (a\pi+b)}{h} \lim_{x\to 0^+} \frac{ah}{h} = a$$

f is differentiable everywhere if $a = -1$. b can be arbitrary constant.

4. Find $\frac{dy}{dx}$ for the following function: **(Answer any one of the following questions only. If you solve both, then first question will be evaluated and the other will be considered unanswered!)**

(a) $\boxed{\text{10 Points}}$ $y = \frac{1}{6}(1 + \cos^2(7x))^3$

Solution:

$$y = \frac{1}{6}u^3$$

where $u = 1 + v$, $v = z^2$, $z = \cos(7x)$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dv}\frac{dv}{dz}\frac{dz}{dx}$$

$$\frac{dy}{du} = \frac{u^2}{2}, \qquad \frac{du}{dv} = 1, \qquad \frac{dv}{dz} = 2z, \qquad \frac{dz}{dx} = -7\sin(7x)$$

$$\begin{aligned} \frac{dy}{dx} &= -7u^2 z \sin(7x) \\ &= -7(1 + \cos^2(7x))^2[\cos(7x)\sin(7x)] \end{aligned}$$

(b) $\boxed{\text{10 Points}}$ $x^4 + \sin y = x^3 y^2$

Solution:

$$\begin{aligned} 4x^3 + y'\cos y &= 3x^2y^2 + 2x^3yy' \\ y'\cos y - 2x^3yy' &= 3x^2y^2 - 4x^3 \\ y' &= \frac{3x^2y^2 - 4x^3}{\cos y - 2x^3 y} \end{aligned}$$

5. (a) 10 Points Find the extreme values (absolute and local) of the function $y = x^3(x-5)^2$ over its natural domain, and state where they occur.

Solution:

$$\begin{aligned} y' =& 3x^2(x-5)^2 + 2x^3(x-5) = (3x^2(x-5) + 2x^3)(x-5) \\ =& (5x^3 - 15x^2)(x-5) = 5x^2(x-3)(x-5) \end{aligned}$$

Critical Points: $x = 0,\ x = 3$ *and* $x = 5$
Extreme Points: Doe to First Derivative Test, the point $(5,0)$ is the local minimum, because the derivative on the left side of the point $(5,0)$ is negative, and it is positive on the other side. Similarly, $(3,108)$ is local maximum, because the derivative on the left side of the point $(3,108)$ is positive, and is the negative on the other side.
Since the sign of the derivative of y does not change at $(0,0)$, it is not an extreme point.

(b) 10 Points Find the value or values of c that satisfy the equation

$$\frac{f(b)-f(a)}{b-a} = f'(c)$$

in the conclusion of the Mean Value Theorem for the function $f(x) = \sqrt{x-1}$ and interval $[1,3]$.

Solution:

$$f'(c) = \frac{1}{2\sqrt{c-1}},\ f(3) = \sqrt{2},\ f(1) = 0$$

$$\frac{1}{2\sqrt{c-1}} = \frac{\sqrt{2}-0}{3-1} = \frac{\sqrt{2}}{2}$$

(c) 15 Points Answer the following questions about the function whose derivative $f'(x) = \dfrac{x^2(x-1)}{x+2},\ x \neq -2$:

(i) What are the critical points of f?

Solution:

$$f'(x) = \frac{x^2(x-1)}{(x+2)} = 0 \Rightarrow x^2 = 0, x-1 = 0, x+2 = 0$$

Critical Points: $x = 0,\ x = 1$. Becaose f is not defined at $x = -2$ it can not be a critical point.

(ii) On what open intervals is f increasing or decreasing?

Solution:

x	$-\infty$ -2	0	1	$+\infty$
$f'(x)$	$+$	$-$	$-$	$+$
$f(x)$	*Increase*	*decrease*	*decrease*	*Increase*

f is increasing on intervals $(-\infty, -2)$ and $(1, \infty)$
f is decreasing on intervals $(-2, 0)$ and $(0, 1)$.

(iii) At what points, if any, does f assume its local maximum and minimum values?

Solution: From First Derivative Test we ca say that;
f has local minimum at $x = 1$ because $f' < 0$ on the left of $x = 1$ and $f' > 0$ on the rigt of $x = 1$.
$x = 0$ is not an extreme point.

Moreover, f has not local maximum at $x = -2$ becaucef f does not defined at $x = -2$