2016–17 **MAT461 Fonksiyonel Analiz I – Bilgi** N. Course

Welcome to *Fonksiyonel Analiz I* aka *Functional Analysis I*.

Course Website

`x.co/mat461`

Kitap/Suggested Text(s)

- Gerald Teschl, *Functional Analysis*, free e-book.
  `www.mat.univie.ac.at/~gerald/ftp/book-fa`

Giriş/Introduction

In MAT221 Linear Algebra I (Lineer Cebir I) and MAT222 Linear Algebra II (Lineer Cebir II) you studied finite dimensional vector spaces and matrices.

Definition. A *vector space* over a field K, is a set X (together with 2 binary operations) that satisfies

$$\mathbf{x}, \mathbf{y} \in X,\ \lambda \in K \qquad \Longrightarrow \qquad \mathbf{x} + \lambda\mathbf{y} \in X.$$

For example, $\mathbb{R}^3$ is a vector space over $\mathbb{R}$ (a "real vector space") and $\mathbb{C}^7$ is a vector space over $\mathbb{C}$ (a "complex vector space").

Definition. Let X and Y be vector spaces over the same field K. A map $A : X \to Y$ is called *linear* iff

$$\mathbf{x}, \mathbf{y} \in X,\ \lambda \in K \qquad \Longrightarrow \qquad A(\mathbf{x} + \lambda\mathbf{y}) = A\mathbf{x} + \lambda A\mathbf{y}.$$

For example,

$$A : \mathbb{R}^3 \to \mathbb{R}^2, \qquad A = \begin{pmatrix} 1 & 1 & 3 \\ 4 & -2 & 0 \end{pmatrix}$$

is linear.

In this course, we are going to study *infinite dimensional* vector spaces and linear maps between them. This is why *Functional Analysis* is also called *Linear Analysis*.

A *functional* is a (linear) map from a vector space X to the field underlying the vector space K. For example;

$$A : \mathbb{R}^3 \to \mathbb{R}, \qquad A\mathbf{x} = x_1 + 2x_2 + 3x_3$$

is a functional.

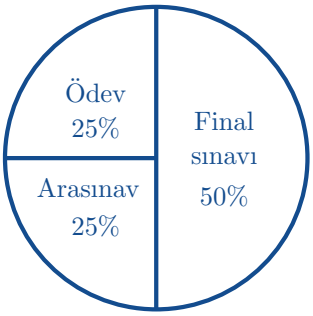
Sometimes we look at vector spaces of functions. For example, $X = \{f : [0,1] \to \mathbb{R} : f \text{ is continuous}\}$ is an infinite dimensional vector space over the field $\mathbb{R}$. The linear map $I : X \to \mathbb{R}$,

$$I(f) = \int_0^1 f(x)\ dx$$

is a functional.

Içerik/Contents

"The only way to learn mathematics is to do mathematics." – Paul Halmos (1916–2006)



During the course, there will be homework problems for you to study. Please remember that your answers to the homework problems must be your own work. Plagiarism is not acceptable: **If you copy another student's homework, or if you allow someone to copy your homework, then you will both receive a mark of zero!** *İntihal bir suçtur: Başka bir öğrencinin ödevinden kopya çekerseniz, ya da sizin ödevinizden kopya çekmesine izin verirseniz, her ikiniz de sıfır alacaksınız!*

There will be only one mid-term exam.

For a course with 3 hours of lectures per week; I expect you to spend atleast 6 hours every week, studying outside of class. At a minimum, you should be reading the textbook, and attempting the exercise questions in there (not just the ones I set for homework).

If you miss a lecture; I expect you to copy your friends' notes or read the textbook, to catch up.

Not/Grades

I will give a pass (grade DD) for a mark of 40/100 or higher, grade DC for ≥ 46, grade CC for ≥ 52, grade CB for ≥ 58, grade BB for ≥ 64, grade BA for ≥ 70, and grade AA for ≥ 76.

Dersler/Lectures

- Çarşamba 9:00–12:00, oda D506

Ofis Saati/Office Hours

If you have any questions, or would like any extra hints for the homework, you can find me in my office at the following time:

- Pazartesi/Monday 16:00–17:00;

Alternately, you can email your questions to me at `neil.course@okan.edu.tr`

Ders programı/Syllabus

- Metric Spaces, Topological Spaces, Banach Spaces, Hilbert Spaces, completeness, bounded operators, sums and quotients of Banach Spaces, spaces of continuous and differentiable functions,
- orthonormal bases, the Projection Theorem, the Riesz Lemma, operators defined via forms, orthogonal sums and tensor products,
- compact operators, the Spectral Theorem for Compact Symmetric Operators, applications to Sturm-Liouville Operators,