OKAN ÜNİVERSİTESİ
MÜHENDİSLİK FAKÜLTESİ
MÜHENDİSLİK TEMEL BİLİMLERİ BÖLÜMÜ

2016–17 **MAT461 Fonksiyonel Analiz I – Ödev 2** N. Course

SON TESLİM TARİHİ: Çarşamba 2 Kasım 2016 saat 10:00'e kadar.

Egzersiz 3 (Norms). [10p] Show that

$$|\|f\| - \|g\|| \leq \|f - g\| .$$

Egzersiz 4 (Banach Spaces). Define

$$\|a\|_\infty := \sup_{n\in\mathbb{N}} |a_n|$$

and

$$\ell^\infty(\mathbb{N}) := \{a = (a_n)_{n=1}^\infty \subseteq \mathbb{C} : \|a\|_\infty < \infty\}.$$

(a) [10p] Show that $\ell^\infty(\mathbb{N})$ is a vector space.

(b) [10p] Show that $\|\cdot\|_\infty$ is a norm on $\ell^\infty(\mathbb{N})$.

(c) [30p] Show that $(\ell^\infty(\mathbb{N}), \|\cdot\|_\infty)$ is a Banach space.

(d) [40p] Show that $\ell^\infty(\mathbb{N})$ is not separable.
[HINT: Consider sequences which take only the value one and zero. How many are there? What is the distance between two such sequences?]

Ödev 1'in çözümleri

1. (a) Since $d(x, y) \leq d(x, z) + d(y, z)$ by definition, we have that $d(x, y) - d(y, z) \leq d(x, z)$. Similarly $-d(x, y) + d(y, z) \leq d(x, z)$, and multiplying by -1 gives $d(x, y) - d(y, z) \geq -d(x, z)$. Therefore $|d(x, y) - d(y, z)| \leq d(x, z)$.

   (b) Using part (a) and the triangle rule for real numbers, we calculate that
   $\left|d(x, y) - d(x', y')\right| = \left|d(x, y) - d(x', y) + d(x', y) - d(x', y')\right| \leq \left|d(x, y) - d(x', y)\right| + \left|d(x', y) - d(x', y')\right|$
   $\leq d(x, x') + d(y, y')$.

   (c) We know that $U \subseteq V \subseteq X$, that $\overline{U} \cap V = V$ (so $\overline{U} \supseteq V$) and that $\overline{V} = X$. So $\overline{\overline{U}} = \overline{V} = X$. But $\overline{\overline{U}} = \overline{U}$, so $\overline{U} = X$ and U is dense in X.

2. (a) $X \setminus \overline{A} = X \cap (\overline{A})^c = X \cap (A^c)^o = (X \cap A^c)^o = (X \setminus A)^o$ and (b) $X \setminus A^o = X \cap (A^o)^c = X \cap \overline{A^c} = \overline{X \cap A^c} = \overline{X \setminus A}$.