OKAN ÜNİVERSİTESİ
MÜHENDİSLİK FAKÜLTESİ
MÜHENDİSLİK TEMEL BİLİMLERİ BÖLÜMÜ

2016–17 **MAT462 Fonksiyonel Analiz II – Ödev 1** N. Course

SON TESLİM TARİHİ: Pazartesi 27 Şubat 2017 saat 17:00'e kadar.

Egzersiz 1 (Nowhere dense). [35p] Let X be a topological space and let $U \subseteq X$ be a subset. Show that

$$U \text{ is closed and nowhere dense} \iff X \setminus U \text{ is open and dense.}$$

[HINT: See Problem 1.4 in Teschl 2015.]

> **Theorem (The Inverse Mapping Theorem).** *Let X and Y be Banach spaces. Let $A \in \mathcal{B}(X, Y)$ be a bounded linear bijection. Then A^{-1} is continuous.*

Egzersiz 2 (The Inverse Mapping Theorem). [30p] Prove the Inverse Mapping Theorem.
[HINT: Read your lecture notes from MAT461, then use the Open Mapping Theorem. This should be an easy question.]

Egzersiz 3 (Closed Operators). [35p] Suppose that

- X and Y are Banach spaces;
- $\mathfrak{D}(A) \subseteq X$;
- $\operatorname{Ran}(A) \subseteq Y$; and
- $A : \mathfrak{D}(A) \to \operatorname{Ran}(A)$ is closed and injective.

Show that $A^{-1} : \operatorname{Ran}(A) \to \mathfrak{D}(A)$ is a closed operator.