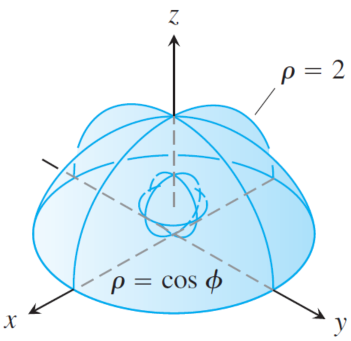
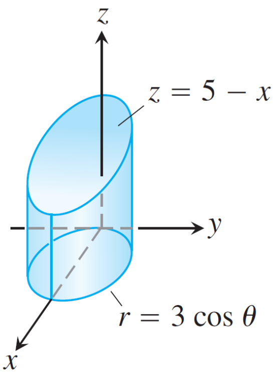
OKAN ÜNİVERSİTESİ
MÜHENDİSLİK-MİMARLIK FAKÜLTESİ
MÜHENDİSLİK TEMEL BİLİMLERİ BÖLÜMÜ

2014–15 **MAT233 Matematik III – Extra Problems** N. Course

!!! This is not homework. Bu ödev değil. !!!





Problem 24 (Spherical Polar Coordinates)**.** Calculate the volume of the solid between the sphere $\rho = \cos\phi$ and the hemisphere $\rho = 2$ for $z \geq 0$.

Problem 25 (Cylindrical Polar Coordinates)**.** Let D be the solid between the surfaces $z = 0$, $r = 3\cos\theta$ and $z = 5 - x$. Define $f : D \to \mathbb{R}$ by $f(x, y, z) = x^2 + y^2$. Calculate

$$\iiint_D f(x, y)\, dV.$$

Problem 26 (Substitutions in Multiple Integrals)**.**

(a) Calculate

$$\int_0^{2/3} \int_y^{2-2y} (x + 2y)e^{(y-x)} dxdy.$$

[HINT: Use the substitution $u = x + 2y$ and $v = x - y$.]

(b) Calculate

$$\int_0^2 \int_{y/2}^{(y+4)/2} y^3(2x - y)e^{(2x-y)^2} dxdy.$$

[HINT: Use the substitution $x = u + \frac{1}{2}v$ and $y = v$.]

Ödev 8'in çözümleri

21. $\iint_R f(x, y)\ dA = \int_0^1 \int_{-x}^{1-x} xy\ dydx = \int_0^1 \left[\frac{1}{2}xy^2\right]_{-x}^{1-x} dx = \int_0^1 \frac{1}{2}x(1-x)^2 - \frac{1}{2}x(-x)^2\ dx$
$= \frac{1}{2}\int_0^1 x - 2x^2\ dx = \frac{1}{2}\left[\frac{1}{2}x^2 - \frac{2}{3}x^3\right]_0^1 = -\frac{1}{12}$.

22. $\int_{x=0}^{x=2} \int_{y=0}^{y=\sqrt{1-(x-1)^2}} \frac{x+y}{x^2+y^2}\ dydx = \int_0^{\frac{\pi}{2}} \int_0^{2\cos\theta} \frac{r\cos\theta + r\sin\theta}{r^2} rdrd\theta = \int_0^{\frac{\pi}{2}} \int_0^{2\cos\theta} \cos\theta + \sin\theta drd\theta$
$= \int_0^{\frac{\pi}{2}} 2\cos^2\theta + 2\cos\theta\sin\theta\ d\theta = \left[\theta + \frac{\sin 2\theta}{2} + \sin^2\theta\right]_0^{\frac{\pi}{2}} = 1 + \frac{\pi}{2}$.

23. We will calculate the volume of the solid in figure (d), then multiply by 8. So
$V = 8\int_{x=0}^{x=1} \int_{y=0}^{y=\sqrt{1-x^2}} \int_{z=0}^{z=\sqrt{1-x^2}} dzdydx = 8\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-x^2}\ dydx = 8\int_0^1 1 - x^2\ dx = \frac{16}{3}$.

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24. $V = \int_0^{2\pi} \int_0^{\pi/2} \int_{\cos\phi}^{2} \rho^2 \sin\phi\ d\rho d\phi d\theta = \ldots = \frac{31\pi}{6}$.

25. Since $f(x, y, z) = r^2$, we have $\iiint_D f(x, y)\ dV = \int_{-\pi/2}^{\pi/2} \int_0^{3\cos\theta} \int_0^{5 - r\cos\theta} r^3\ dzdrd\theta = \ldots = \frac{729\pi}{32}$

26. (a) Since $J(u, v) = -\frac{1}{3}$, we have $\int_0^{2/3} \int_y^{2-2y} (x + 2y)e^{(y-x)} dxdy = \int_0^2 \int_0^u ue^{-v} \left|-\frac{1}{3}\right| dvdu = \ldots = \frac{1}{3}(3e^{-2} + 1)$.

(b) Since $J(u, v) = 1$, we have $\int_0^2 \int_{y/2}^{(y+4)/2} y^3(2x - y)e^{(2x-y)^2} dxdy = \int_0^2 \int_0^2 v^3(2u)e^{4u^2} dudv = \ldots = e^{16} - 1$.