2014–15 **MAT233 Matematik III – Ödev 1** N. Course

SON TESLİM TARİHİ: Çarşamba 8 Ekim 2014 saat 10:00'e kadar.

Egzersiz 1 (Parabolas)**.** [2 × 20p] Find each parabola's focus and directrix. Then draw a sketch of the parabola. Include the focus and directrix in your sketch.

(a) $y^2 = 12x$.

(b) $x^2 = -8y$.

Egzersiz 2 (Ellipses)**.**

(a) [20p] If

$$\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a,$$

show that

$$\frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1.$$

[HINT: Start by writing $\sqrt{(x+c)^2 + y^2} = 2a - \sqrt{(x-c)^2 + y^2}$ and then square both sides.]

(b) [20p] Draw a sketch of the ellipse $3x^2 + 2y^2 = 6$. Include the foci in your sketch.

Egzersiz 3 (Hyperbolas)**.** [20p] A hyperbola has foci at $F_1 = (0, \sqrt{2})$ and $F_2 = (0, -\sqrt{2})$, and has asymptotes $y = x$ and $y = -x$. Find the equation of the hyperbola.

NOTE: For 1(a), 1(b) and 2(b); when I ask you to draw a graph, please make sure that your drawing is clear and of a good size. I don't want to see tiny graphs.

Some formulae from MAT111 and MAT112

$$\cos^2\theta + \sin^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$
$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$
$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$\cosh^2\theta - \sinh^2\theta = 1$$
$$\sinh 2\theta = 2\sinh\theta\cosh\theta$$
$$\cosh 2\theta = \cosh^2\theta + \sinh^2\theta$$
$$\cosh^2\theta = \frac{1}{2}(\cosh 2\theta + 1)$$
$$\sinh^2\theta = \frac{1}{2}(\cosh 2\theta - 1)$$

$$\cos 0 = \cos 0^\circ = 1$$
$$\sin 0 = \sin 0^\circ = 0$$
$$\cos\tfrac{\pi}{4} = \cos 45^\circ = \tfrac{1}{\sqrt{2}}$$
$$\sin\tfrac{\pi}{4} = \sin 45^\circ = \tfrac{1}{\sqrt{2}}$$
$$\cos\tfrac{\pi}{3} = \cos 60^\circ = \tfrac{1}{2}$$
$$\sin\tfrac{\pi}{3} = \sin 60^\circ = \tfrac{\sqrt{3}}{2}$$
$$\cos\tfrac{\pi}{2} = \cos 90^\circ = 0$$
$$\sin\tfrac{\pi}{2} = \sin 90^\circ = 1$$

$$(uv)' = uv' + u'v$$
$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$
$$(f\circ g)'(x) = f'(g(x))\,g'(x)$$
$$\left(f^{-1}\right)'(x) = \frac{1}{f'\left(f^{-1}(x)\right)}$$
$$y' = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$
$$\frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt}$$
$$\int u\ dv = uv - \int v\ du$$

$$\tfrac{d}{dx}x^n = nx^{n-1}$$
$$\tfrac{d}{dx}\sin x = \cos x$$
$$\tfrac{d}{dx}\cos x = -\sin x$$

$\tan x = \frac{\sin x}{\cos x}$ $\qquad \frac{d}{dx}\tan x = \sec^2 x$ $\qquad \int \tan x\ dx = \ln|\sec x| + C$

$\sec x = \frac{1}{\cos x}$ $\qquad \frac{d}{dx}\sec x = \sec x\tan x$ $\qquad \int \sec x\ dx = \ln|\sec x + \tan x| + C$

$\cot x = \frac{\cos x}{\sin x}$ $\qquad \frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$ $\qquad \int \cot x\ dx = \ln|\sin x| + C$

$\operatorname{cosec} x = \frac{1}{\sin x}$ $\qquad \frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x\cot x$ $\qquad \int \operatorname{cosec} x\ dx = -\ln|\operatorname{cosec} x + \cot x| + C$

$$\tfrac{d}{dx}\sin^{-1}\tfrac{x}{a} = \tfrac{1}{\sqrt{a^2-x^2}}$$
$$\tfrac{d}{dx}\tan^{-1}\tfrac{x}{a} = \tfrac{a}{a^2+x^2}$$
$$\tfrac{d}{dx}\sec^{-1}\tfrac{x}{a} = \tfrac{a}{|x|\sqrt{x^2-a^2}}$$

$\sinh x = \frac{e^x - e^{-x}}{2}$ $\qquad \frac{d}{dx}\sinh x = \cosh x$

$\cosh x = \frac{e^x + e^{-x}}{2}$ $\qquad \frac{d}{dx}\cosh x = \sinh x$

$$\tfrac{d}{dx}e^x = e^x$$
$$\tfrac{d}{dx}\ln|x| = \tfrac{1}{x}$$

$$V = \int_a^b A(x)\ dx$$
$$V = \int_a^b \pi[R(x)]^2\ dx$$
$$V = \int_a^b \pi\left([R(x)]^2 - [r(x)]^2\right)\ dx$$
$$V = \int_a^b 2\pi\binom{\textbf{shell}}{\textbf{radius}}\binom{\textbf{shell}}{\textbf{height}}\ dx$$
$$L = \int ds$$
$$S = \int_a^b 2\pi y\ ds \quad \textbf{or} \quad S = \int_a^b 2\pi x\ ds$$
$$ds = \sqrt{dx^2 + dy^2}$$

$$\sum_{k=1}^{n} k = \frac{1}{2}n(n+1)$$
$$\sum_{k=1}^{n} k^2 = \frac{1}{6}n(n+1)(2n+1)$$
$$\sum_{k=1}^{n} k^3 = \left(\frac{1}{2}n(n+1)\right)^2$$