2014–15 **MAT233 Matematik III – Ödev 2** N. Course

SON TESLİM TARİHİ: Çarşamba 15 Ekim 2014 saat 10:00'e kadar.

Egzersiz 4 (Eccentricity)**.** [25p] A conic section has vertices at $(\pm 70, 0)$ and has eccentricity 0.1. Find a Cartesian equation for this conic section. (What type of conic section is this?)

Egzersiz 5 (Rotating Coordinate Axes)**.**

(a) [40p] Rotate the coordinate axes to change $x^2 + xy + y^2 = 1$ into an equation that has no cross product (xy or $x'y'$) term.
[HINT: First solve $\cot 2\alpha = \frac{A-C}{B}$ to find the angle of rotation α.]

(b) [15p] Then sketch the curve.

Egzersiz 6 (The Discriminant Test)**.**

(a) [10p] What is the discriminant of $3x^2 - 5xy + 3y^2 - x + 2y = 0$? Is this curve an ellipse, a parabola or a hyperbola?

(b) [10p] What is the discriminant of $x^2 - 6xy + 9y^2 - 6x + 9y = -12$? Is this curve an ellipse, a parabola or a hyperbola?

Ödev 1'in çözümleri

1. (a) Focus is $(3, 0)$, directrix is $x = -3$.
(b) Focus is $(0, -2)$, directrix is $y = 2$. Sketches omitted.

2. (a) First $\sqrt{(x+c)^2 + y^2} = 2a - \sqrt{(x-c)^2 + y^2}$. Squaring gives $(x+c)^2 + y^2 = 4a^2 - 4a\sqrt{(x-c)^2 + y^2} + (x-c)^2 + y^2$. So $4cx = 4a^2 - 4a\sqrt{(x-c)^2 + y^2}$ which rearranges to $a\sqrt{(x-c)^2 + y^2} = a^2 - cx$. Squaring again, we get $a^2(x-c)^2 + a^2y^2 = a^4 - 2a^2cx + c^2x^2$, which is $(a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2)$. Finally divide by $a^2(a^2 - c^2)$ to get $\frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1$.
(b) $c = \sqrt{a^2 - b^2} = \sqrt{3 - 2} = 1$. The foci are $(0, \pm 1)$. The vertices are $(0, \pm\sqrt{3})$. The ellipse crosses the x-axis at $(\pm\sqrt{2}, 0)$. Sketch omitted.

3. The foci are at $(0, \pm c) = (0, \pm\sqrt{2})$ so $c = \sqrt{2}$. The asymptotes are $y = \pm\frac{a}{b}x = \pm x$ so $a = b$. Since $2 = c^2 = a^2 + b^2 = 2a^2$, we know that $a = b = 1$. Therefore the equation is $y^2 - x^2 = 1$.