OKAN ÜNİVERSİTESİ
MÜHENDİSLİK-MİMARLIK FAKÜLTESİ
MÜHENDİSLİK TEMEL BİLİMLERİ BÖLÜMÜ

2014–15 **MAT233 Matematik III – Ödev 7** N. Course

SON TESLİM TARİHİ: Çarşamba 10 Aralık 2014 saat 10:00'e kadar.

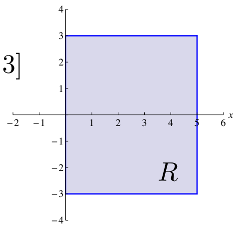
Egzersiz 18 (Local Extrema and Saddle Points). [30p] Let $f : \mathbb{R}^2 \to \mathbb{R}$,

$$f(x, y) = 2x^3 - y^3 + 3x^2 + 3y^2 - 2.$$

Find all the local minima, local maxima and saddle points of f.

Egzersiz 19 (Absolute Extreme Values). [40p] Let

$$R := \{(x, y) \in \mathbb{R}^2 : \ 0 \le x \le 5, \ -3 \le y \le 3\} = [0, 5] \times [-3, 3]$$



and let $g : R \to \mathbb{R}$,

$$g(x, y) = x^2 + xy - y^2 - 6x.$$

Find the absolute maximum and absolute minimum of g on R.

Egzersiz 20 (Lagrange Multipliers). [30p] Find the maximum value and minimum value of

$$f(x, y, z) = 10x - 4y + 2z$$

on the sphere $x^2 + y^2 + z^2 = 30$.

Ödev 6'nın çözümleri

13. If (x, y) is a point on the line, then $\mathbf{T}(x, y) = (x - x_0)\mathbf{i} + (y - y_0)\mathbf{j}$ is a vector parallel to the line. So $0 = \mathbf{T} \cdot \mathbf{N} = A(x - x_0) + B(y - y_0)$.

14. We calculate that $f_x = -6xz$ so $f_{xx} = -6z$; $f_y = -6yz$ so $f_{yy} = -6z$; and $f_z = 6z^2 - 3x^2 - 3y^2$ so $f_{zz} = 12z$. Therefore $f_{xx} + f_{yy} + f_{zz} = 0$.

15. $\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} = \left[\frac{\left(\frac{1}{y}\right)}{\left(\frac{x}{y}\right)^2+1}\right]\cos v + \left[\frac{\left(\frac{-x}{y^2}\right)}{\left(\frac{x}{y}\right)^2+1}\right]\sin v = \frac{y\cos v}{x^2+y^2} - \frac{x \sin v}{x^2+y^2} = \frac{(u\sin v)(\cos v)-(u\cos v)(\sin v)}{u^2} = 0$ and $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v} = \left[\frac{\left(\frac{1}{y}\right)}{\left(\frac{x}{y}\right)^2+1}\right](-u\sin v + \left[\frac{\left(\frac{-x}{y^2}\right)}{\left(\frac{x}{y}\right)^2+1}\right](u\cos v) = -\frac{yu\sin v}{x^2+y^2} - \frac{xu\cos v}{x^2+y^2} = \frac{-(u\sin v)(u\sin v)-(u\cos v)(u\cos v)}{u^2} = -\sin^2 v - \cos^2 v = -1$.

16. Since $\frac{\partial g}{\partial x} = \frac{2x}{x^2+y^2}$ and $\frac{\partial g}{\partial y} = \frac{2y}{x^2+y^2}$, we have that $\nabla g|_{(1,1)} = g_x(1,1)\mathbf{i} + g_y(1,1)\mathbf{j} = \mathbf{i} + \mathbf{j}$.

17. First $\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{\mathbf{i}+\mathbf{j}+\mathbf{k}}{\sqrt{1^2+1^2+1^2}} = \frac{\mathbf{i}}{\sqrt{3}} + \frac{\mathbf{j}}{\sqrt{3}} + \frac{\mathbf{k}}{\sqrt{3}}$. Since $h_x = 2x$, $h_y = 4y$ and $h_z = -6z$, we have that $(D_{\mathbf{u}}h)_{P_0} = \nabla h|_{(1,1,1)} \cdot \mathbf{u} = (2\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}) \cdot \left(\frac{\mathbf{i}}{\sqrt{3}} + \frac{\mathbf{j}}{\sqrt{3}} + \frac{\mathbf{k}}{\sqrt{3}}\right) = 2\left(\frac{1}{\sqrt{3}}\right) + 4\left(\frac{1}{\sqrt{3}}\right) - 6\left(\frac{1}{\sqrt{3}}\right) = 0$.