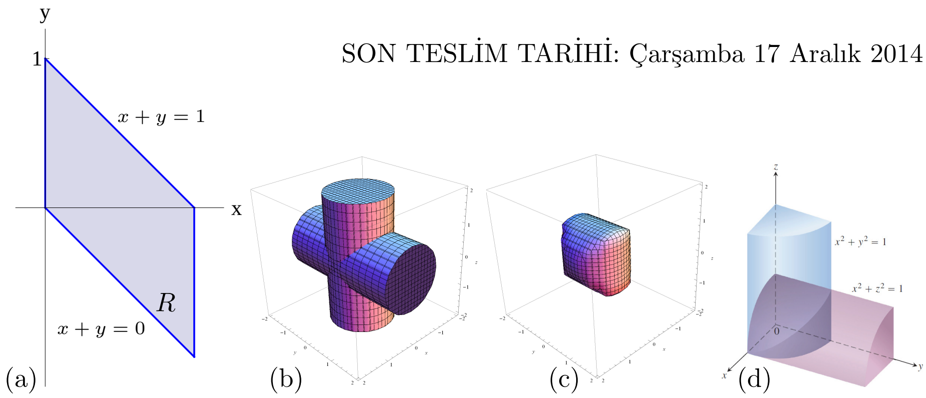
OKAN ÜNİVERSİTESİ
MÜHENDİSLİK-MİMARLIK FAKÜLTESİ
MÜHENDİSLİK TEMEL BİLİMLERİ BÖLÜMÜ

2014–15 **MAT233 Matematik III – Ödev 8** N. Course

SON TESLİM TARİHİ: Çarşamba 17 Aralık 2014 saat 10:00'e kadar.



Egzersiz 21 (Double Integrals)**.** [30p] Let R be the region bounded by the lines $x = 0$, $x = 1$, $x + y = 0$ and $x + y = 1$ (see (a) above) and let $f(x, y) = xy$. Calculate $\iint_R f(x, y)\, dA$.

Egzersiz 22 (Double Integrals in Polar Coordinates)**.** [30p] Use polar coordinates to calculate

$$\int_{x=0}^{x=2} \int_{y=0}^{y=\sqrt{1-(x-1)^2}} \frac{x+y}{x^2+y^2}\, dydx.$$

[HINT: $dydx = rdrd\theta$.] [HINT: Try plotting the polar curve $r = 2\cos\theta$ first, e.g. http://x.co/5yzR4]

Egzersiz 23 (Volumes using Triple Integrals)**.** [40p] Let D_1 be the cylinder $\{(x, y, z) : x^2 + y^2 \le 1\}$ and let D_2 be the cylinder $\{(x, y, z) : x^2 + z^2 \le 1\}$ (see (b) above). Let $D = D_1 \cap D_2$ (see (c) above). Figure (d) shows $\frac{1}{8}$ of D. Calculate the volume of D.

Ödev 7'nin çözümleri

18. There are no boundary points, so we just need to find the critical points of f. $0 = f_x = 6x^2 + 6x \implies x = 0$ or $x = -1$, and $0 = f_y = -3y^2 + 6y \implies y = 0$ or $y = 2$. The critical points are $(0, 0)$, $(-1, 0)$, $(0, 2)$ and $(-1, 2)$.

    Next we need to calculate the Hessian at these points. $f_{xx} = 12x + 6$, $f_{xy} = 0$ and $f_{yy} = -6y + 6$, so $H(f) = f_{xx}f_{yy} - f_{xy}^2 = (12x + 6)(6 - 6y)$.

    Since $H(f)(0, 0) = 36 > 0$, and $f_{xx}(0, 0) = 6 > 0$, the critical point $(0, 0)$ is a local minimum. Since $H(f)(-1, 0) = -36 < 0$, the critical point $(-1, 0)$ is a saddle point. Since $H(f)(0, 2) = -36 < 0$, the critical point $(0, 2)$ is a saddle point. Finally, since $H(f)(-1, 2) = 36 > 0$ and $f_{xx}(-1, 2) = -6 > 0$, the critical point $(-12, 2)$ is a local maximum.

19. First we look for critical points in the interior of R. $0 = g_x = 2x+y-6$ and $0 = g_y = x-2y \implies (x, y) = (\frac{12}{5}, \frac{6}{5}) \in R$, and $g(\frac{12}{5}, \frac{6}{5}) = -\frac{36}{5}$.

    Next we must look at the boundary of R. Let $A = (0, -3)$, $B = (0, 3)$, $C = (5, 3)$ and $D = (5, -3)$ be the four corners of R.

    (i) On the line segment AB we have $x = 0$, so consider $h(y) = g(0, y) = -y^2$ for $-3 \le y \le 3$. $0 = h'(y) = -2y \implies y = 0$. So we calculate $g(0, 0) = h(0) = 0$, $g(0, -3) = h(-3) = -9$ and $g(0, 3) = h(3) = -9$ (Note: don't forget the endpoints!).

    (ii) On BC we have $y = 3$, so consider $k(x) = g(x, 3) = x^2 - 3x - 9$ for $0 \le x \le 5$. $0 = k'(x) = 2x - 3 \implies x = \frac{3}{2}$. So we calculate $g(\frac{2}{3}, 3) = -\frac{95}{9}$ and $g(5, 3) = k(5) = 1$ (Note: we have already calculated $g(0, 3) = k(0)$).

    (iii) On CD we have $x = 5$, so consider $l(y) = g(5, y) = -y^2+5y-5$ for $-3 \le y \le 3$. $0 = l'(y) = -2y+5 \implies y = \frac{5}{2}$. So we calculate $g(5, \frac{5}{2}) = l(\frac{5}{2}) = \frac{5}{4}$ and $g(5, -3) = l(-3) = -29$.

    (iv) Finally, on AD we have $y = -3$. so consider $m(x) = g(x, -3) = x^2 - 9x - 9$ for $0 \le x \le 5$. $0 = m'(x) = 2x - 9 \implies x = \frac{9}{2}$. So we calculate $g(\frac{9}{2}, -3) = m(\frac{9}{2}) = -\frac{117}{4}$.

    We have calculated the values $-\frac{36}{5}, 0, -9, -9, -\frac{95}{9}, 1, \frac{5}{4}, -29, -\frac{117}{4}$. So the absolute maximum of g is $\frac{5}{4}$ which g attains at $(5, \frac{5}{2})$. The absolute minimum of g is $-\frac{117}{4}$ which g attains at $(\frac{9}{2}, -3)$.

20. Set $g(x, y, z) = x^2 + y^2 + z^2 - 30$. We need to find $x, y, z, \lambda \in \mathbb{R}$ such that $\nabla f = \lambda \nabla g$ and $g = 0$. Since $10\mathbf{i} - 4\mathbf{j} + 2\mathbf{k} = \nabla f = \lambda \nabla g = s\lambda x\mathbf{i} + 2\lambda y\mathbf{j} + 2\lambda z\mathbf{k}$, we have $10 = 2\lambda x$, $-4 = 2\lambda y$ and $2 = 2\lambda z$. So $z = \frac{1}{\lambda}$, $y = -\frac{2}{\lambda} = -2z$ and $x = \frac{5}{\lambda} = 5z$. So $0 = g(x, y, z) = g(5z, -2z, z) = 25z^2 + 4z^2 + z^2 - 30 \implies z = \pm 1$. Therefore $(x, y, z) = (5, -2, 1)$ or $(-5, 2, -1)$. Since $f(5, -2, 1) = 60$ and $f(-5, 2, -1) = -60$, we see that the maximum value of f on the sphere is 60, and the minimum value is -60