



SON TESLİM TARİHİ: Salı 29 Eylül 2015 saat 15:00'e kadar.

Egzersiz 1 (Pool Party).

English

Your swimming pool contains 150,000 litres of water. It has been contaminated by 5 kg of a dye that leaves a swimmer's skin an unattractive green.

The swimming pool's filtering system can take water from the pool, remove the dye, and return the water to the pool at a rate of 500 litres/minute.

You have invited your friends to a pool party that is scheduled to begin 4 hours later. If and only if the concentration of the dye is less than 0.005 grams/litre, then the swimming pool is safe to swim in.

Türkçe

Yüzme havuzunuz 150.000 litre su almaktadır. Havuza, 5 kg boya karışmış ve havuza girenlerin cildini itici bir yeşile boyamaktadır.

Yüzme havuzunun filtre sistemi 500 litre/dakika hızla havuzdan suyu alıp boyadan temizleyerek suyu havuza geri vermektedir.

Arkadaşlarınızı havuz partisine davet etmişsiniz ve parti 4 saat sonra başlayacak. Eğer boyanın yoğunluğu 0,005 gram/litre'den azsa havuz suyu, içinde yüzmek için güvenli demektir.

- (a) [20p] Write down an initial value problem for the amount of dye in the swimming pool.
(You must state if your time t is measured in seconds, minutes or hours. You must also state which units [kilograms? grams? ounces? tonnes? stones?] you are using for the amount of dye in the swimming pool.)
- (b) [20p] Solve the initial value problem that you wrote in part (a).
- (c) [5p] How much green dye is in the swimming pool after 1 hour?
- (d) [15p] Is your swimming pool's filtering system capable of reducing the dye concentration to a safe level in 4 hours, or should you cancel your party? (Justify your answer.)

Egzersiz 2 (Direction Fields). Consider the differential equation

$$\frac{dy}{dt} = t - y.$$

- (a) [30p] Draw a direction field for this differential equation.
- (b) [10p] Describe the behaviour of $y(t)$ as $t \rightarrow \infty$. (Does this behaviour depend on the value of y at $t = 0$? Is y asymptotic to a line as $t \rightarrow \infty$?)