2015–16 **MAT371 Diferansiyel Denklemler – Ödev 2** N. Course

SON TESLİM TARİHİ: Salı 13 Ekim 2015 saat 15:00'e kadar.

Egzersiz 3 (Classification). [$6 \times (2+2)$p] For each of the following differential equations; give the order of the equation and state whether the equation is linear or non-linear. The first one is done for you.

(ω) $y\frac{d^2y}{dt^2} - t\frac{d^3y}{dt^3} = \frac{dy}{dt}$ (3rd order, non-linear)

(a) $\frac{dy}{dt} + ty = 0$

(b) $\frac{d^4y}{dt^4} + \frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} + \frac{dy}{dt} + y^2 = \log t$

(c) $\frac{d^3y}{dt^3} + \sin t = y$

(d) $t\frac{dy}{dt} + (\cos^2 t)y = t^3 + \frac{d^2y}{dt^2}$

(e) $t^2\frac{d^2y}{dt^2} + t\frac{dy}{dt} + 2y = \sin y$

(f) $(1+y^2)\frac{d^2y}{dt^2} + t\frac{dy}{dt} + y = e^t$

Egzersiz 4 (Integrating Factors). Consider the inital value problem

$$\begin{cases} \frac{dy}{dt} + 2y = 2te^{2t} \\ y(0) = 2. \end{cases}$$

(a) [4p] Find the required integrating factor $\mu(t)$.

(b) [4p] Multiply the differential equation by $\mu(t)$.

(c) [4p] Rearrange the left-hand side to the form $\frac{d}{dt}[\mu(t)y]$.

(d) [15p] Integrate this equation to find the general solution to the differential equation.

(e) [9p] Find the solution to the initial value problem.

Egzersiz 5 (Separable ODEs).

(a) [20p] Solve the initial value problem, using the fact that the equation is separable,

$$\begin{cases} \frac{dy}{dx} = (1-2x)y^2 \\ y(0) = -\frac{1}{6}. \end{cases}$$

(b) [10p] Plot the graph of the solution.

(c) [10p] Determine the interval in which the solution is defined.

Ödev 1'in çözümleri

1. (a) Let t denote time in minutes, and let $D(t)$ denote the amount (in grams) of dye in the pool at time t. Clearly $D(0) = 5000$.

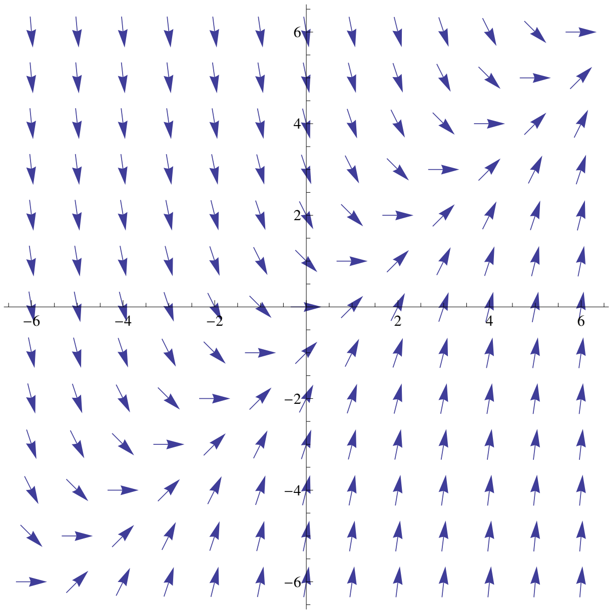
   The concentration of dye in the pool is $\frac{D(t)}{150000}$ grams/litre. Every minute the filtering system cleans 500 litres. In other words, every minute the filtering system removes $500 \times \frac{D(t)}{150000} = \frac{D(t)}{300}$ grams of dye from the pool.

   Therefore, the required IVP is
$$\begin{cases} \frac{dD}{dt} = -\frac{D(t)}{300} \\ D(0) = 5000 \end{cases}$$

   (b) Rearranging $\frac{dD}{dt} = -\frac{D(t)}{300}$ gives $\frac{dD}{D} = -\frac{dt}{300}$. Integrating gives $\log|D| = -\frac{t}{300} + c$, and then rearranging gives $D(t) = Ce^{-\frac{t}{300}}$ (for a different constant C). Finally, we use the initial condition to see that $5000 = D(0) = Ce^0 = C$. Hence $D(t) = 5000e^{-\frac{t}{300}}$.

   (c) After 1 hour, $t = 60$ and $D(60) = 5000e^{-\frac{60}{300}} \approx 4094$ grams.

   (d) After 4 hours, $t = 240$ and $D(240) = 5000e^{-\frac{240}{300}} \approx 2247$ grams. This gives a concentration of $\approx \frac{2247}{150000} \approx 0.015$ grams/litre which is too high. CANCEL THE PARTY!!!!!



2. (a)

   (b) y is asymptotic to the line $y = (t-1)$ as $t \to \infty$ [only 5 pts for "$y \to \infty$"].