2015–16 **MAT371 Diferansiyel Denklemler – Ödev 3** N. Course

SON TESLİM TARİHİ: Pazar 20 Ekim 2013 saat 15:00'e kadar.

Egzersiz 6 (Using Theorem 1). [4×5p] Find (without solving the problem) an interval (α, β) in which the solution of the given initial value problem is certain to exist.

(a) $(t-5)y' + (\log t)y = 4t$, $y(1) = 2$,

(b) $y' + (\tan t)y = \sin t$, $y(\pi) = 0$,

(c) $(4-t^2)y' + 2ty = 3t^2$, $y(3) = 1$,

(d) $(4-t^2)y' + 2ty = 3t^2$, $y(-1) = -3$.

Egzersiz 7 (A Non-linear Equation). Consider

$$\begin{cases} \frac{dy}{dt} = -ty(y-3) \\ y(0) = y_0. \end{cases}$$

(a) [20p] Draw a direction field for the differential equation.

(b) [20p] Sketch (roughly draw) several solutions of the differential equation. [You do not need to solve the differential equation.]

Egzersiz 8 (Understanding Theorem 2).

(a) [16p] Check that both $y_1(t) = -t^2/4$ and $y_2(t) = 1-t$ are solutions of the initial value problem

$$\begin{cases} y' = \frac{-t+\sqrt{t^2+4y}}{2} \\ y(2) = -1. \end{cases}$$

(b) [14p] Where are these solutions valid?

(c) [10p] Explain why the existence of two solutions of the given problem does not contradict the uniqueness part of Theorem 2.

Ödev 2'nin çözümleri

3. (a) 1st order, linear, (b) 4th order, non-linear (c) 3rd order, linear (d) 2nd order, linear (e) 2nd order, non-linear (f) 2nd order, non-linear

4. (a) $\mu(t) = e^{2t}$. (b) $e^{2t}y' + 2e^{2t}y = 2te^{4t}$. (c) $\frac{d}{dt}\left[e^{2t}y\right] = 2te^{4t}$. (d) $e^{2t}y = \int 2te^{4t}\,dt = \frac{1}{2}te^{4t} - \int \frac{1}{2}e^{4t}\,dt = \frac{1}{2}te^{4t} - \frac{1}{8}e^{4t} + C$, so $y = Ce^{-2t} + \frac{1}{2}(t-\frac{1}{4})e^{2t}$. (e) $y = \frac{17}{8}e^{-2t} + \frac{1}{2}(t-\frac{1}{4})e^{2t}$.

5. (a) Rearrange to $\frac{dy}{y^2} = (1-2x)dx$. Then integrate to get $-\frac{1}{y} = x - x^2 + C$, and rearrange to $y = 1/(x^2-x-C)$. To satisfty $y(0) = -\frac{1}{6}$ we must have $y = 1/(x^2-x-6)$ (b) below (c) $-2 < x < 3$.