

N. Course

SON TESLİM TARİHİ: Salı 24 Kasım 2015 saat 10:00'e kadar.

Egzersiz 11 (Exact ODEs).  $[4 \times 5p]$  Are the following ODEs exact or not exact? [40p] If the ODE is exact, then find the solution [If the ODE is not exact, then you do not need to solve it].

- (a) (2x+3) + (2y-2)y' = 0,
- (b) (2x+4y) + (2x-2y)y' = 0,
- (c)  $\frac{dy}{dx} = -\frac{ax-by}{bx-cy}$ ,  $a, b, c \in \mathbb{R}, b \neq 0$ , (d)  $\frac{dy}{dx} = -\frac{ax+by}{bx+cy}$ ,  $a, b, c \in \mathbb{R}, b \neq 0$ .

Egzersiz 12 (Exact ODEs). Consider

$$y + (2x - ye^y)\frac{dy}{dx} = 0.$$

- (a) [10p] Is this equation exact  $(M_y = N_x)$ ?
- (b) [5p] Multiply the equation by the integrating factor  $\mu(x,y) = y$ . Show that the equation is now exact.
- (c) [25p] Now solve the equation that you wrote in (b). [HINT: First, you need to find  $\psi$  such that  $\psi_x = \mu M$  and  $\psi_y = \mu N$ . Then the solutions are given by  $\psi(x, y) = c$ .]



10. (a) y = 0 is unstable, y = 1 is asymptotically stable. (b)  $y(t) = \frac{y_0}{y_0 + (1-y_0)e^{-\alpha t}}$ . (c)  $\lim_{t \to \infty} y(t) = \lim_{t \to \infty} \frac{y_0}{y_0 + (1-y_0)e^{-\alpha t}} = \frac{y_0}{y_0 + 0} = 1$ .