SON TESLİM TARİHİ: Çarşamba 2 Aralık 2015 saat 12:00'e kadar.

Egzersiz 13 (Homogeneous second order linear ODE).

(a) [40p] Solve

$$\begin{cases} 6y'' - 5y' + y = 0, \\ y(0) = 4, \\ y'(0) = 0. \end{cases}$$

(b) [10p] Sketch the graph of the solution.

(c) [5p] How does the solution behave as $t \to \infty$?

Egzersiz 14 (Fundamental sets of solutions). [3×15p] In each of the following problems; (i) verify that the functions y_1 and y_2 are solutions of the given differential equation, (ii) do they form a fundamental set of solutions? The first one is done for you.

(ω) $x^2y'' - x(x+2)y' + (x+2)y = 0$, $x > 0$, $\qquad y_1(x) = x$, $y_2(x) = xe^x$.

Solution: (i) Since

$$x^2y_1'' - x(x+2)y_1' + (x+2)y_1 = x^2(0) - x(x+2)(1) + (x+2)x = 0$$

and

$$\begin{aligned} x^2y_2'' - x(x+2)y_2' + (x+2)y_2 &= x^2(xe^x + 2e^x) - x(x+2)(xe^x + e^x) + (x+2)xe^x \\ &= e^x(x^3 + 2x^2 - x^3 - 2x^2 - x^2 - 2x + x^2 + 2x) = 0, \end{aligned}$$

both y_1 and y_2 solve the equation. (ii) Since

$$W(y_1, y_2)(x) = \begin{vmatrix} y_1(x) & y_2(x) \\ y_1'(x) & y_2'(x) \end{vmatrix} = \begin{vmatrix} x & xe^x \\ 1 & xe^x + e^x \end{vmatrix} = x^2e^x + xe^x - xe^x = x^2e^x \neq 0$$

for $x > 0$, the answer is **yes** – y_1 and y_2 do form a fundamental set of solutions.

(a) $y'' + 4y = 0$, $\qquad y_1(t) = \cos 2t$, $y_2(t) = \sin 2t$.

(b) $y'' - 2y' + y = 0$, $\qquad y_1(t) = 2e^t$, $y_2(t) = 7e^t$.

(c) $(1 - x\cot x)y'' - xy' + y = 0$, $\; 0 < x < \pi$, $\qquad y_1(x) = x$, $y_2(x) = \sin x$.

Ödev 5'in çözümleri

11. (a) Exact. $x^2 + 3x + y^2 - 2y = c$. (b) Not exact. (c) Not exact. (d) Exact. $ax^2 + 2bxy + cy^2 = k$.

12. (a) No, $M_y = 1$ but $N_x = 2$. (b) $y^2 + (2xy - y^2e^y)y' = 0$. $(\mu M)_y = 2y = (\mu N)_x$. (c) We need to find ψ such that $\psi_x = y^2$ and $\psi_y = 2xy - y^2e^y$. Integrating the first equation, we get $\psi = \int \psi_x \, dx + h(y) = xy^2 + h(y)$ for some function h. Differentiating, we get $\psi_y = 2xy + h'(y)$. We need $h'(y) = -y^2e^y$, so we choose $h(y) = -(y^2 - 2y + 2)e^y$. Therefore $\psi(x, y) = xy^2 - (y^2 - 2y + 2)e^y$ and the solution to the ODE is $xy^2 - (y^2 - 2y + 2)e^y = c$ for any constant $c \in \mathbb{R}$.