2015–16 **MAT371 Diferansiyel Denklemler – Ödev 7** N. Course

SON TESLİM TARİHİ: Çarşamba 9 Aralık 2015 saat 11:30'e kadar.

Egzersiz 15 (Homogeneous second order linear ODE). [3 × 20p] Find the general solution of the following ODEs:

(a) $y'' + 5y' = 0$,

(b) $y'' - 2y' + 6y = 0$,

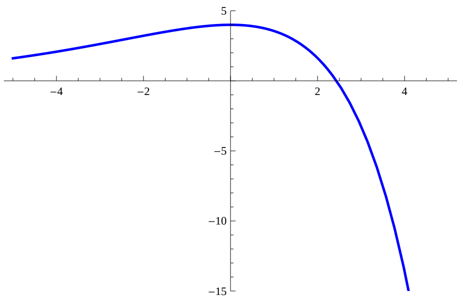
(c) $y'' - 2y' + y = 0$.

Egzersiz 16 (Inhomogeneous second order linear ODE). [40p] Solve

$$\begin{cases} y'' - 2y' + y = te^{2t} + 4\cos t, \\ y(0) = 1, \\ y'(0) = 1. \end{cases}$$

Ödev 6'nın çözümleri

13. (a) The characteristic equation is $0 = 6r^2 - 5r + 1 = (3r-1)(2r-1)$. So $r_1 = \frac{1}{3}$ and $r_2 = \frac{1}{2}$. So the general solution to the ODE is $y(t) = c_1 e^{t/3} + c_2 e^{t/2}$. Using $y(0) = 4$ and $y'(0) = 0$ we get $c_1 + c_2 = 4$ and $\frac{c_1}{3} + \frac{c_2}{2} = 0$. So $c_1 = 12$ and $c_2 = -8$. Therefore the solution to the IVP is $y(t) = 12e^{t/3} - 8e^{t/2}$. (b) below (c) $y \to -\infty$ as $t \to \infty$.



14. (a) Yes. (b) No. (c) Yes.

2nd Order Homogeneous Linear ODEs with Constant Coefficients

Consider

$$ay'' + by' + cy = 0. \tag{4}$$

The *characteristic equation* of (4) is

$$ar^2 + br + c = 0. \tag{5}$$

The roots of (5) are

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

CASE 1: $(b^2 - 4ac > 0)$ $r_1, r_2 \in \mathbb{R}$ and $r_1 \neq r_2$.
The general solution of (4) is

$$y(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t}.$$

CASE 2: $(b^2 - 4ac = 0)$ $r_1, r_2 \in \mathbb{R}$ and $r_1 = r_2$.
The general solution of (4) is

$$y(t) = c_1 e^{r_1 t} + c_2 t e^{r_1 t}.$$

CASE 3: $(b^2 - 4ac < 0)$ $r_1, r_2 \in \mathbb{C} \setminus \mathbb{R}$. So $r_1 = \lambda + i\mu$, $r_2 = \lambda - i\mu$ $(\lambda, \mu \in \mathbb{R})$.
The general solution of (4) is

$$y(t) = c_1 e^{\lambda t} \cos \mu t + c_2 e^{\lambda t} \sin \mu t.$$