2015–16 **MAT371 Diferansiyel Denklemler – Ödev 8** N. Course

SON TESLİM TARİHİ: Çarşamba 16 Aralık 2015 saat 11:30'e kadar.

Egzersiz 17 (Systems of Equations). [40p] Solve

$$\begin{cases} \frac{dx}{dt} = 5x - y \\ \frac{dy}{dt} = 3x + y \\ x(0) = 2 \\ y(0) = -1. \end{cases}$$

Egzersiz 18 (Systems of Equations). Let

$$A = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix}.$$

(a) [30p] Find the general solution of $\mathbf{x}' = A\mathbf{x}$.

(b) [30p] Draw a phase plot of $\mathbf{x}' = A\mathbf{x}$.

Ödev 7'nin çözümleri

15. (a) The characteristic polynomial of $y'' + 5y' = 0$ is $0 = r^2 + 5r = r(r+5)$ so $r = 0, -5$. So the general solution of the ODE is $y = c_1 + c_2 e^{-5t}$.
(b) The characteristic polynomial of $y'' - 2y' + 6y = 0$ is $0 = r^2 - 2r + 6$. So $r = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{2 \pm \sqrt{4-24}}{2} = 1 \pm i\sqrt{5}$. We have complex roots, so the general solution of the ODE is $y = c_1 e^t \cos\sqrt{5}t + c_2 e^t \sin\sqrt{5}t$.
(c) The characteristic polynomial of $y'' - 2y + y = 0$ is $0 = r^2 - 2r + r = (r-1)^2$ so $r = 1$. We have repeated roots, so the general solution of the ODE is $y = c_1 e^t + c_2 t e^t$.

16. From the previous exercise, we know that the general solution of the homogeneous equation is $y = c_1 e^t + c_2 t e^t$. First we consider $y'' - 2y' + y = te^{2t}$ and the ansatz $Y(t) = (At + B)e^{2t}$. Then $Y'' - 2Y' + Y = (2At + 2A + B)e^{2t}$ so $A = 1$ and $B = -2$. So $Y(t) = (t-2)e^{2t}$.
Next we consider $y'' - 2y' + y = 4\cos t$. We use an ansatz of $Y(t) = A\cos t + B\sin t$. Then $Y'' - 2Y' + Y = 2A\sin t - 2B\cos t$ so $A = 0$ and $B = -2$. Therefore $Y(t) = -2\sin t$.
So the general solution of the inhomogenous equation is $y(t) = c_1 e^t + c_2 t e^t + e^{2t}(t-2) - 2\sin t$.
Finally we use the initial conditions to find c_1 and c_2. In this case, $y(t) = 3e^t + 3te^t + e^{2t}(t-2) - 2\sin t$.