

17. First we rewrite the problem as a matrix equation,

$$\mathbf{x}' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}.$$

The eigenvalues of the matrix are 2 and 4, with corresponding eigenvectors of $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

So the general solution is $\mathbf{x}(t) = c_1 \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t}$. To satisfy the initial condition we must have $\mathbf{x}(t) = -\frac{3}{2} \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{2t} + \frac{7}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t}$.

Therefore

$$x(t) = -\frac{3}{2}e^{2t} + \frac{7}{2}e^{4t} \quad \text{and} \quad y(t) = -\frac{9}{2}e^{2t} + \frac{7}{2}e^{4t}.$$

18. (a) $\mathbf{x}(t) = c_1 \begin{pmatrix} 1 \\ -4 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$.

