

Formula Page

$$\begin{aligned} \cos \theta &= \sin \left(\frac{\pi}{2} - \theta \right) \\ \cos^2 \theta + \sin^2 \theta &= 1 \\ 1 + \tan^2 \theta &= \sec^2 \theta \\ 1 + \cot^2 \theta &= \operatorname{cosec}^2 \theta \\ \cos(A + B) &= \cos A \cos B - \sin A \sin B \\ \sin(A + B) &= \sin A \cos B + \cos A \sin B \\ \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ \sin 2\theta &= 2 \sin \theta \cos \theta \\ \cos^2 \theta &= \frac{1}{2}(1 + \cos 2\theta) \\ \sin^2 \theta &= \frac{1}{2}(1 - \cos 2\theta) \\ c^2 &= a^2 + b^2 - 2ab \cos \theta \end{aligned}$$

$$\begin{aligned} \cos 0 &= \cos 0^\circ = 1 \\ \sin 0 &= \sin 0^\circ = 0 \\ \cos \frac{\pi}{4} &= \cos 45^\circ = \frac{1}{\sqrt{2}} \\ \sin \frac{\pi}{4} &= \sin 45^\circ = \frac{1}{\sqrt{2}} \\ \cos \frac{\pi}{3} &= \cos 60^\circ = \frac{1}{2} \\ \sin \frac{\pi}{3} &= \sin 60^\circ = \frac{\sqrt{3}}{2} \\ \cos \frac{\pi}{2} &= \cos 90^\circ = 0 \\ \sin \frac{\pi}{2} &= \sin 90^\circ = 1 \end{aligned}$$

$$\begin{aligned} (uv)' &= uv' + u'v \\ \left(\frac{u}{v} \right)' &= \frac{u'v - uv'}{v^2} \\ (f \circ g)'(x) &= f'(g(x))g'(x) \\ (f^{-1})'(x) &= \frac{1}{f'(f^{-1}(x))} \\ \int u \, dv &= uv - \int v \, du \\ \frac{d}{dt} f(x(t), y(t)) &= \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} \end{aligned}$$

$$\frac{d}{dx} x^n = nx^{n-1}$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\int \tan x \, dx = \log |\sec x| + C$$

$$\sec x = \frac{1}{\cos x}$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\int \sec x \, dx = \log |\sec x + \tan x| + C$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$$

$$\int \cot x \, dx = \log |\sin x| + C$$

$$\operatorname{cosec} x = \frac{1}{\sin x}$$

$$\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$$

$$\int \operatorname{cosec} x \, dx = -\log |\operatorname{cosec} x + \cot x| + C$$

$$\frac{d}{dx} \sin^{-1} \frac{x}{a} = \frac{1}{\sqrt{a^2 - x^2}}$$

$$\frac{d}{dx} \tan^{-1} \frac{x}{a} = \frac{a}{a^2 + x^2}$$

$$\frac{d}{dx} \sec^{-1} \frac{x}{a} = \frac{a}{|x|\sqrt{x^2 - a^2}}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\frac{d}{dx} \sinh x = \cosh x$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\frac{d}{dx} \cosh x = \sinh x$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \log |x| = \frac{1}{x}$$

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Soru 1 (Exact Equations) Consider

$$(2xye^y + 2ye^{-x^2}) + (e^y + ye^y + 2xe^{-x^2}) \frac{dy}{dx} = 0 \quad (1)$$

This equation is of the form $M(x, y) + N(x, y)y' = 0$.

(a) [4p] Is this equation exact?

(b) [4p] Show that

$$\frac{M_y - N_x}{N} = 2x.$$

(c) [12p] Find an integrating factor $\mu(x)$ that solves

$$\frac{d\mu}{dx} = \left(\frac{M_y - N_x}{N} \right) \mu$$

$$(2xye^y + 2ye^{-x^2}) + (e^y + ye^y + 2xe^{-x^2}) \frac{dy}{dx} = 0 \quad (1)$$

- (d) [1p] Multiply (1) by the integrating factor that you found in part (c). [This new equation will be called (2).]

(2)

- (e) [4p] Show that (2) is exact?

[HINT: If (2) is not exact, then your answer to part (c) is probably wrong.]

- (f) [25p] Solve (2).

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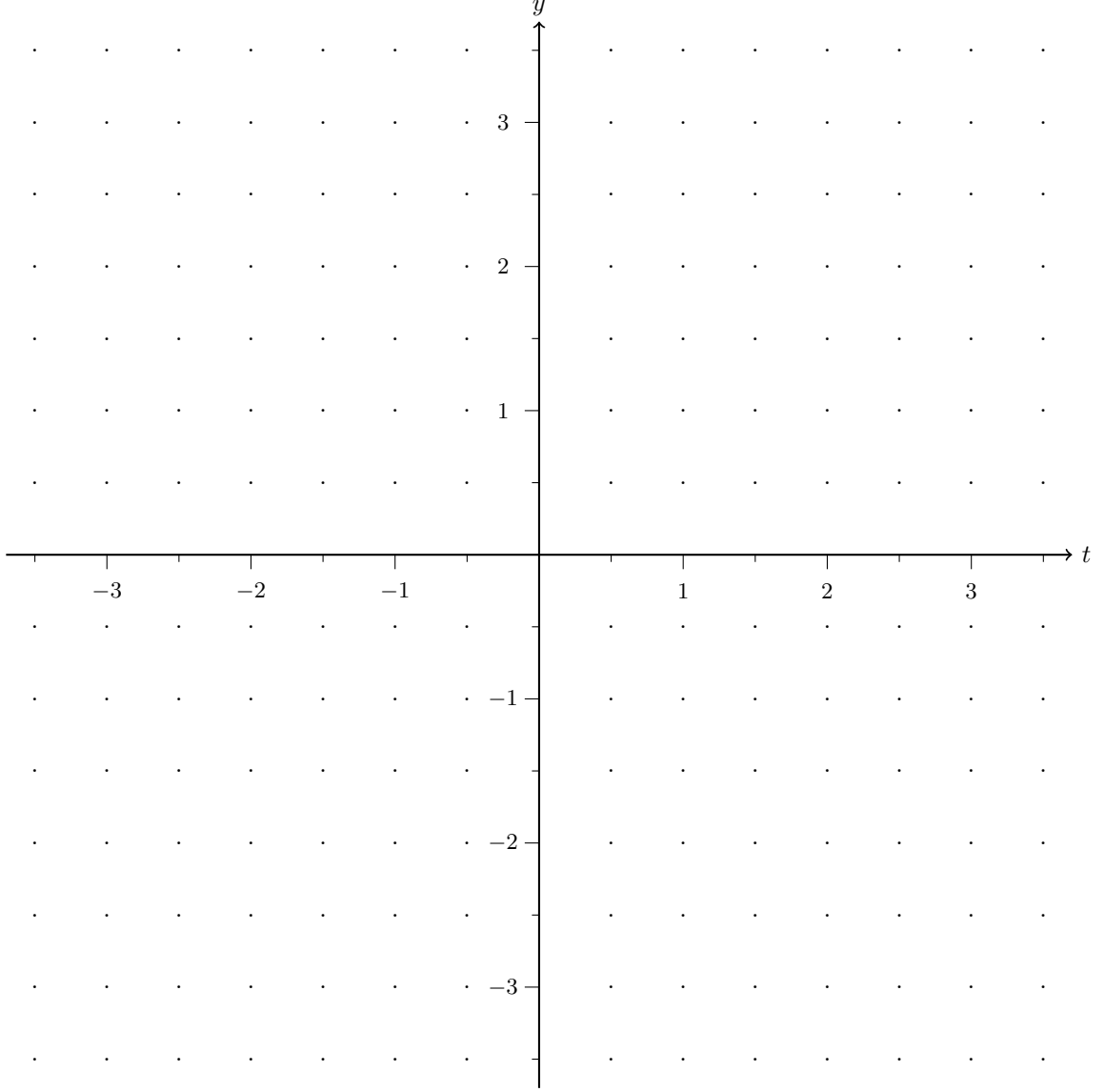
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Soru 2 (a)

(a) [25p] Draw a direction field for the equation

$$\frac{dy}{dt} + y = \sin \frac{\pi t}{2}. \quad (3)$$

[HINT: I want to see 225 arrows – one on every dot, and one on every mark on the axes.]



(b) [20p] Find the general solution to

$$\frac{dy}{dt} + y = \sin \frac{\pi t}{2}. \quad (3)$$

(c) [5p] Solve

$$\begin{cases} \frac{dy}{dt} + y = \sin \frac{\pi t}{2} \\ y(0) = 3. \end{cases}$$

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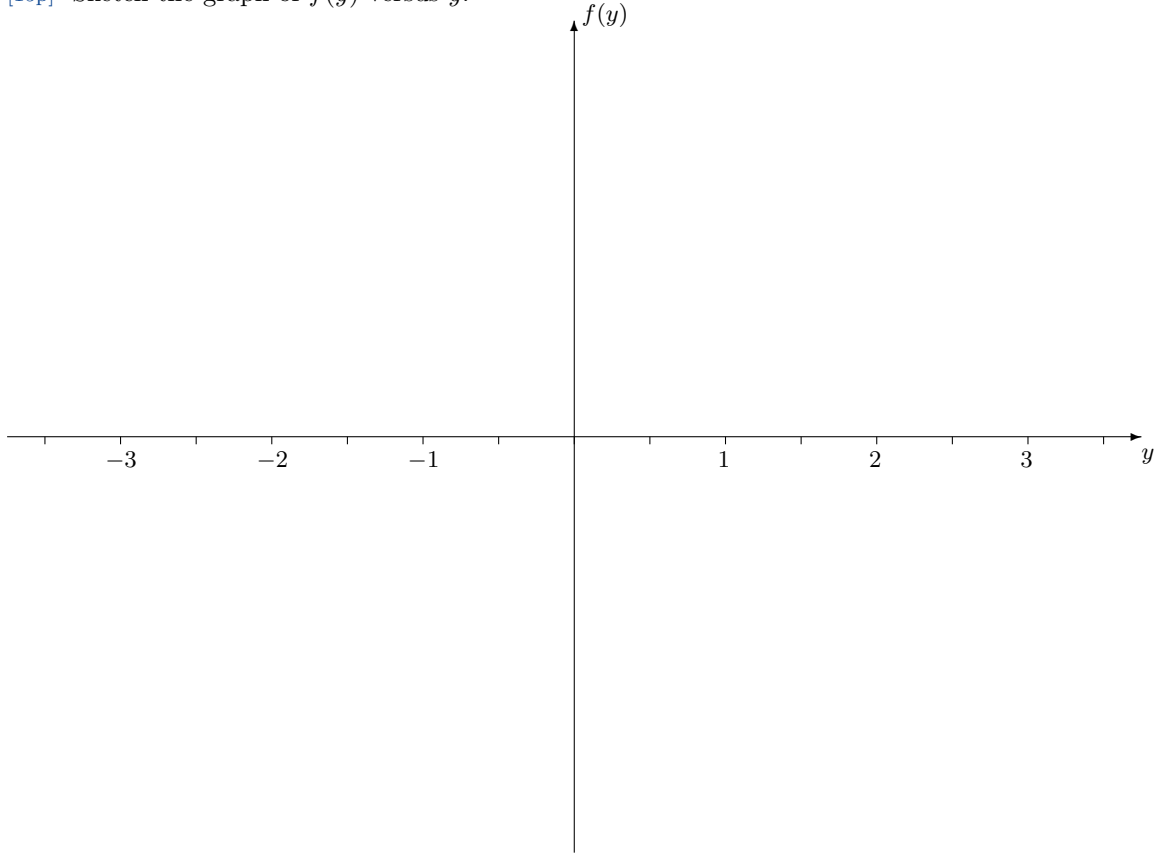
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Soru 3 (Autonomous Equations) Consider the autonomous differential equation

$$\frac{dy}{dt} = f(y) = y^2(4 - y^2). \quad (4)$$

(a) [10p] Find all of the critical points of (4).

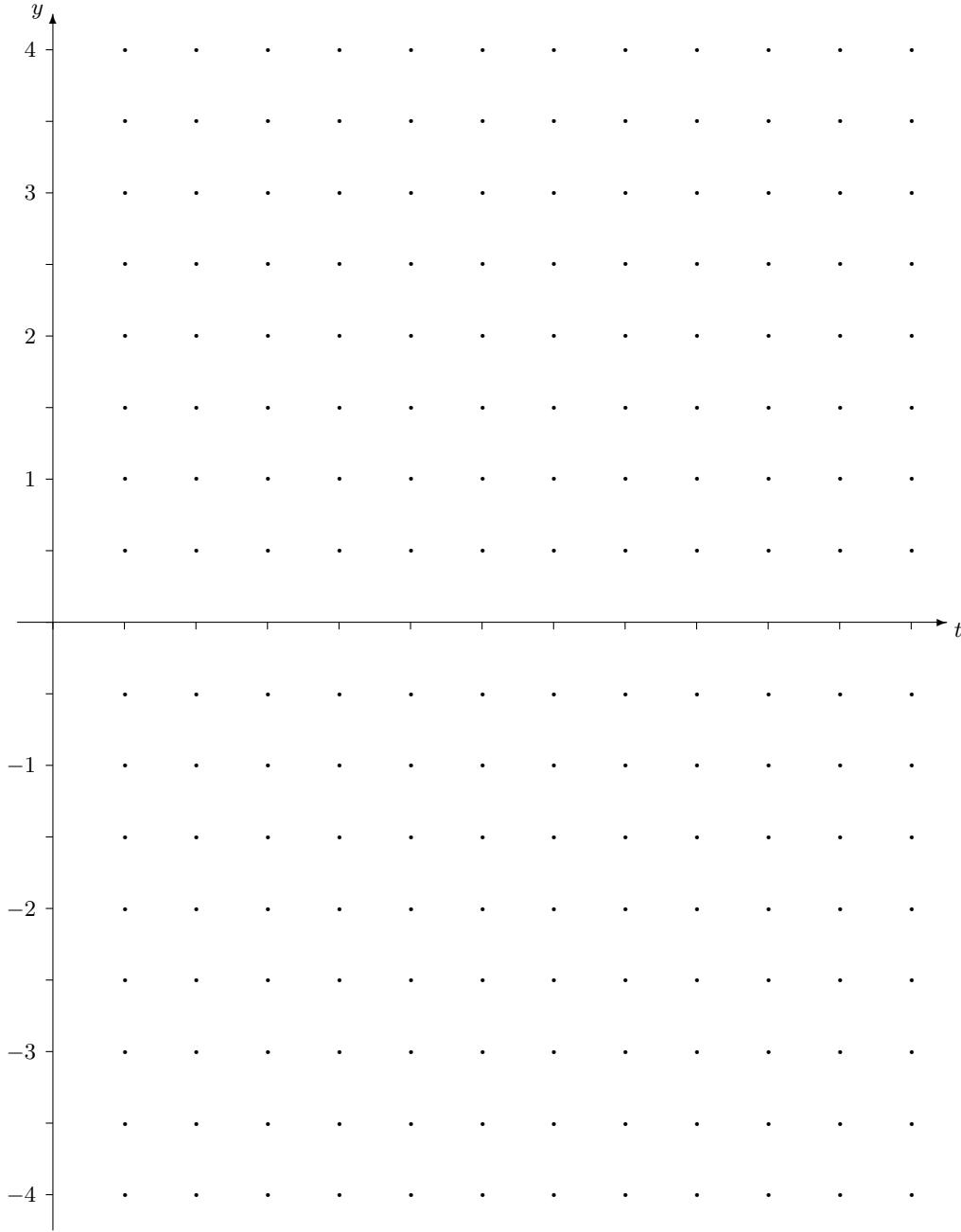
(b) [15p] Sketch the graph of $f(y)$ versus y .



(c) [9p] Determine whether each critical point is asymptotically stable, unstable or semistable.

$$\frac{dy}{dt} = f(y) = y^2(4 - y^2). \tag{4}$$

(d) [16p] Sketch 10 (or more) different solutions of (4).



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