OKAN ÜNİVERSİTESİ
MÜHENDİSLİK-MİMARLIK FAKÜLTESİ
MÜHENDİSLİK TEMEL BİLİMLERİ BÖLÜMÜ

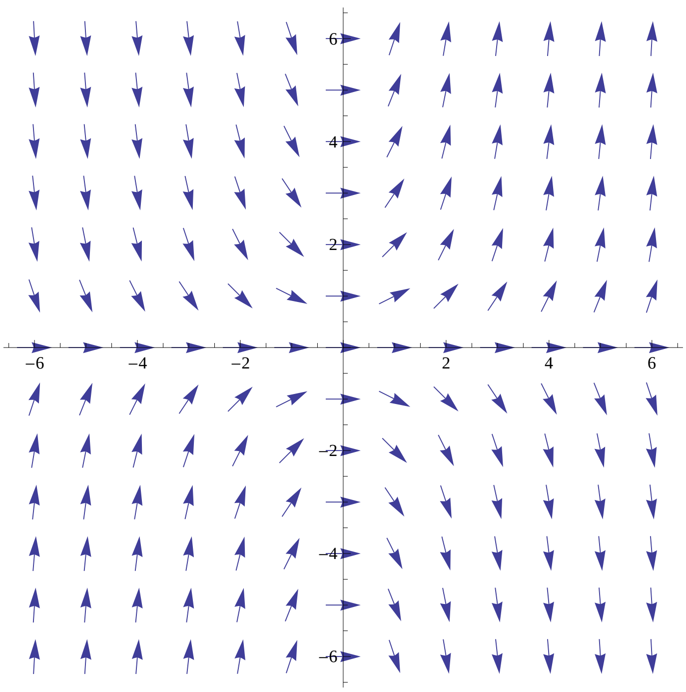
2015.11.10 **MAT371 Diferansiyel Denklemler – Ara Sınavın Çözümleri** N. Course

**Soru 1 (Separable ODEs).**

(a) [25p] Draw a direction field for the equation

$$\frac{dy}{dx} = \frac{xy}{2}. \tag{1}$$

[HINT: I want to see 169 arrows – one on every dot, and one on every mark on the axes.]




(b) [15p] Find the general solution to

$$\frac{dy}{dx} = \frac{xy}{2}. \tag{1}$$

First we separate the variables

$$\frac{dy}{dx} = \frac{xy}{2}$$
$$\frac{dy}{y} = \frac{1}{2}x\ dx$$

Then we integrate

$$\int \frac{dy}{y} = \int \frac{1}{2}x\ dx$$
$$\log|y| = \frac{1}{4}x^2 + c$$

Rearranging gives

$$y(x) = Ae^{\frac{x^2}{4}}$$

for some constant $A \in \mathbb{R}$.

(c) [5p] Solve

$$\begin{cases} \frac{dy}{dx} = \frac{xy}{2} \\ y(1) = -7. \end{cases}$$

Using the initial condition, we can see that

$$-7 = y(1) = Ae^{\frac{1^2}{4}} = Ae^{\frac{1}{4}}$$

Therefore $A = -7e^{-\frac{1}{4}}$. Hence the solution is

$$y(x) = -7e^{\frac{x^2-1}{4}}.$$

(d) [5p] Let $y(x)$ denote your answer to part (c). Calculate $y'(x)$ and $\frac{1}{2}xy(x)$.

Differentiating $y(x) = -7e^{\frac{x^2-1}{4}}$ gives

$$y'(x) = -7(\frac{2x}{4})e^{\frac{x^2-1}{4}} = -\frac{7}{2}xe^{\frac{x^2-1}{4}}.$$

Moreover

$$\frac{1}{2}xy(x) = \frac{1}{2}x\left(-7e^{\frac{x^2-1}{4}}\right) = -\frac{7}{2}xe^{\frac{x^2-1}{4}} = y'(x).$$

**Soru 2 (Autonomous Equations).** Consider the autonomous differential equation

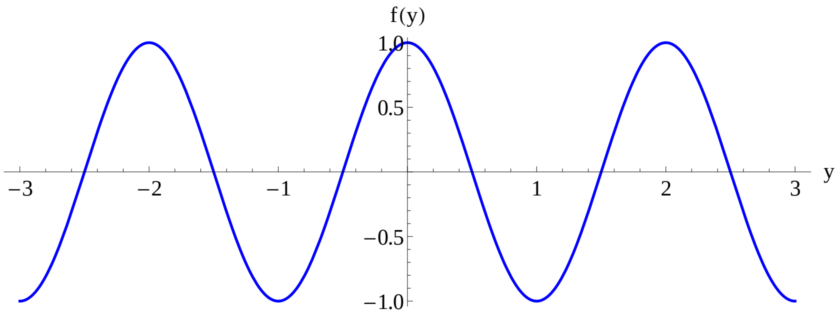
$$\frac{dy}{dt} = \cos(\pi y). \tag{2}$$

This equation is of the form $y' = f(y)$, with $f(y) = \cos(\pi y)$.

(a) [10p] Find all of the critical points of (2).

$y = \frac{1}{2} + n$ for all $n \in \mathbb{Z}$.

(b) [10p] Sketch the graph of $f(y)$ versus $y$.

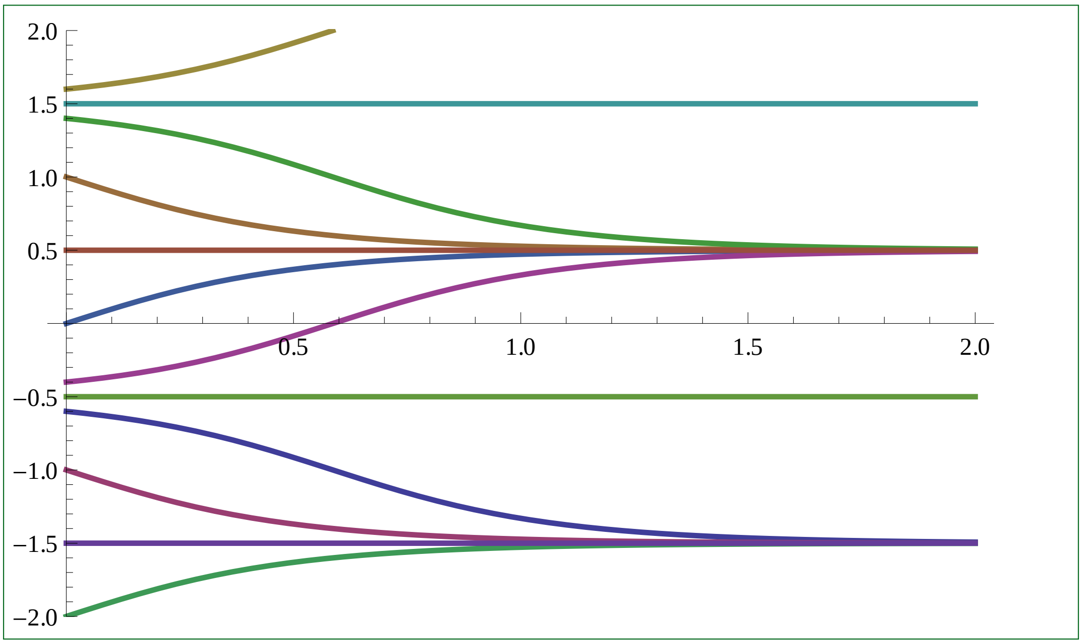



(c) [10p] Determine whether each critical point is asymptotically stable, unstable or semistable.

$y = 2n - \frac{1}{2}$ is unstable and $y = 2n + \frac{1}{2}$ is asymptotically stable.

$$\frac{dy}{dt} = \cos(\pi y) \tag{2}$$

(d) [20p] Sketch 10 (or more) different solutions of (2).




**Soru 3 (Linear Equations).**

(a) [10p] Calculate

$$\frac{d}{dt}\left(-\frac{1}{2}e^{-t}(\sin t+\cos t)\right).$$

$$\begin{aligned}\frac{d}{dt}\left(-\frac{1}{2}e^{-t}(\sin t+\cos t)\right)&=\frac{1}{2}e^{-t}(\sin t+\cos t)-\frac{1}{2}e^{-t}(\cos t-\sin t)\\&=e^{-t}\sin t\end{aligned}$$

Suppose that the differentiable function $y:[0,\infty)\to\mathbb{R}$ satisfies the following 3 conditions:

(a) $y'-y=1+3\sin t$

(b) $y(0)=y_0$

(c) $-\infty<\lim\limits_{t\to\infty}f(t)<\infty$

(b) [40p] Find $y_0\in\mathbb{R}$.

First we solve the ODE using the integrating factor $\mu(t)=e^{-t}$ and part (a):

$$\begin{aligned}y'-y&=1+3\sin t\\e^{-t}y'-e^{-t}y&=e^{-t}+3e^{-t}\sin t\\\left(e^{-t}y\right)'&=e^{-t}+3e^{-t}\sin t\\e^{-t}y&=\int e^{-t}+3e^{-t}\sin t\,dt=-e^{-t}-\frac{3}{2}e^{-t}(\sin t+\cos t)+c\\y&=-1-\frac{3}{2}(\sin t+\cos t)+ce^{t}\end{aligned}$$

To get $-\infty<\lim_{t\to\infty}f(t)<\infty$, we must have that $c=0$. But then

$$y_0=y(0)=-1-\frac{3}{2}(\sin 0+\cos 0)=-1-\frac{3}{2}(0+1)=-\frac{5}{2}.$$

For $f$ to satisfy all three conditions, we must have $y_0=-\frac{5}{2}$.