2015–16 **MAT372 K.T.D.D. – Ödev 2** N. Course

SON TESLİM TARİHİ: Çarşamba 24 Şubat 2016 saat 12:00'e kadar.

Egzersiz 4.

(a) [15p] Show that $u(x,y) = f(e^{-x}y)$ is a solution of $u_x + yu_y = 0$, for any differentiable function f.

(b) [25p] Show that

$$\begin{cases} u_x + yu_y = 0 \\ u(x,0) = 1 \end{cases}$$

has infinitely many solutions.

Egzersiz 5 (Classification). [6×10p] Classify each of the following as hyperbolic, parabolic or elliptic at every point (x,y) of the domain. The first one is done for you.

(ω) $2u_{xx} - xu_{yy} = 0$, domain= $\mathbb{R}^2$.
Solution: Since $A = 2$, $B = 0$ and $C = -x$, if follows that $\Delta = B^2 - 4AC = 8x$. Therefore the PDE is hyperbolic for $x > 0$, parabolic for $x = 0$ and elliptic for $x < 0$.

(a) $xu_{xx} - u_{yy} = x^2$, domain= $\mathbb{R}^2$;

(b) $x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} = e^x$, domain= $\mathbb{R}^2$;

(c) $e^xu_{xx} - e^yu_{yy} = u$, domain= $\mathbb{R}^2$;

(d) $xu_{xx} + u_{xy} - u_{yy} = 0$, domain= $\{(x,y) \in \mathbb{R}^2 : x \leq 0\}$;

(e) $y^2u_{xx} - 3xyu_{xy} + x^2u_{yy} + xyu_x + y^2u_y = 0$, domain= $\mathbb{R}^2$;

(f) $\frac{1}{4}u_{xx} + (\cos x)u_{xy} + (\frac{1}{2}\cos(2x) + 1)u_{yy} = \cos y$, domain= $\mathbb{R}^2$.

Ödev 1'in çözümleri

1. (a) Third order, (b) Second order, (c) Fifth order, (d) Second order, (e) Second order.

2. (a) Linear, nonhomogeneous, (b) Linear, nonhomogeneous, (c) Quasilinear, nonhomogeneous, (d) Nonlinear, nonhomogeneous, (e) Linear, nonhomogeneous (f) Quasilinear, homogeneous, (g) Nonlinear, nonhomogeneous, (h) Quasilinear, homogeneous (i) Quasilinear, homogeneous, (j) Linear, homogeneous.

3. $x^2u_{xx} - y^2u_{yy} = x^2\left(y^2F'' + \frac{y^2}{x^3}G''\right) - y^2\left(x^2F'' + \frac{1}{x}G''\right) = 0$.