2015–16 **MAT372 K.T.D.D. – Ödev 3** N. Course

SON TESLİM TARİHİ: Çarşamba 9 Mart 2016 saat 12:00'e kadar.

Egzersiz 6. Consider

$$u_{xy} = xy \tag{1}$$

[20p] Find the general solution $u(x, y)$ to (1).

Egzersiz 7 (Canonical Forms). Consider

$$u_{xx} + 5u_{xy} + 4u_{yy} + 7u_y = \sin x \tag{2}$$

(a) [5p] Is (2) a hyperbolic PDE, a parabolic PDE, or an elliptic PDE?

(b) [10p] Find the characteristic equation of (2).

(c) [15p] Find the characteristic curve(s) of (2).

(d) [15p] Sketch the graph(s) of the characteristic curve(s) of (2).

(e) [35p] Find a canonical form for (2).

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\begin{aligned}
&Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = G\\
&\quad A^* = A\xi_x^2 + B\xi_x\xi_y + C\xi_y^2\\
&\quad B^* = 2A\xi_x\eta_x + B(\xi_x\eta_y + \xi_y\eta_x) + 2C\xi_y\eta_y\\
&\quad C^* = A\eta_x^2 + B\eta_x\eta_y + C\eta_y^2\\
&\quad D^* = A\xi_{xx} + B\xi_{xy} + C\xi_{yy} + D\xi_x + E\xi_y\\
&\quad E^* = A\eta_{xx} + B\eta_{xy} + C\eta_{yy} + D\eta_x + E\eta_y\\
&\quad F^* = F\\
&\quad G^* = G\\
&\quad H^* = -D^*u_\xi - E^*u_\eta - F^*u + G^*
\end{aligned}
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Ödev 2'nin çözümleri

4. (a) $u_x + yu_y = e^{-x}yf'(e^{-x}y) + y(e^{-x}f'(e^{-x}y)) = 0$
(b) We can satisfy the boundary condition with any function f which satisfies $f(0) = 1$. For example, $f(z) = cz + 1$ for any $c \in \mathbb{R}$. Therefore the problem has infinitely many solutions.

5. (a) $A = x$, $B = 0$, and $C = -1$ so $\Delta = 4x$. The PDE is hyperbolic for $x > 0$, parabolic for $x = 0$ and elliptic for $x < 0$. (b) Parabolic, (c) Elliptic, (d) Hyperbolic for $0 \geq x > -\frac{1}{4}$, parabolic for $x = -\frac{1}{4}$, and elliptic for $x < -\frac{1}{4}$, (e) hyperbolic, (f) ($B^2 - 4AC = -\frac{1}{2}$) elliptic.