OKAN ÜNİVERSİTESİ
MÜHENDİSLİK-MİMARLIK FAKÜLTESİ
MÜHENDİSLİK TEMEL BİLİMLERİ BÖLÜMÜ

2015–16 | **MAT372 K.T.D.D. – Ödev 5** | N. Course

SON TESLİM TARİHİ: Çarşamba 6 Nisan 2016 saat 12:00'e kadar.

NEW RULE: Poor quality photos of answers sent by email will no longer be accepted.

I prefer to receive your answers on paper. If you must email your answers, then you must either

(i) prepare them with LaTeX;

(ii) use a word processor;

(iii) write them on paper, then use a proper flatbed scanner to scan them; or

(iv) write them on paper, then use a "scanner" app on your mobile phone to scan them.

Make sure that your name and student number are clearly visible on every page that you send.

Egzersiz 9 (Fan-like Characteristics and Shock Waves). Consider the PDE

$$\frac{\partial u}{\partial t} + \frac{1}{2}u\frac{\partial u}{\partial x} = 0, \qquad 0 < t < 3 \tag{1}$$

subject to the initial condition.

$$u(x,0) = \begin{cases} 1 & x < 1 \\ 3 & 1 < x < 5 \\ 1 & x > 5. \end{cases} \tag{2}$$

(a) [10p] Replace (1) by a system of 2 ODEs.

(b) [20p] Plot the characteristics $\left(\overset{t}{\uparrow}\!\!\rightarrow_{x}\right)$ of this problem.

(c) [50p] Solve (1) subject to (2).

(d) [4p+8p+8p] Sketch the graph $\left(\overset{u}{\uparrow}\!\!\rightarrow_{x}\right)$ of the solution, $u(x,t)$, at times $t = 0$, $t = 1$ and $t = 2$.

Ödev 4'ün çözümleri

8. (a) Since $\Delta = B^2 - DAC = 0$, the PDE is parabolic. The characteristic equation is $\frac{dy}{dx} = \frac{B\pm\sqrt{\Delta}}{2A} = \frac{2\pm0}{2} = 1$ and the characteristic curve is $y = x + c$. Using the transformation $\xi = y - x$ and $\eta = y$ we find the canonical form $u_{\eta\eta} = u + 7$.

   Keeping ξ constant, this is a second order linear ODE in η. You know how to solve equations like this from MAT371. Its general solution is $u(\xi,\eta) = C_1(\xi)e^{\eta} + C_2(\xi)e^{-\eta} + 7$.

   Finally, changing back to the original variables, we get $u(x,y) = C_1(y-x)e^{y} + C_2(y-x)e^{-y} - 7$.

   (b) $u_{xx} + 2u_{xy} + u_{yy} - u = (C_1''e^{y} + C_2''e^{-y}) + 2(-C_1''e^{y} - C_1'e^{y} - C_2''e^{-y} + C_2'e^{-y}) + (C_1''e^{y} + 2C_1'e^{y} + C_1e^{y} + C_2''e^{y} - 2C_2'e^{-y} + C_2e^{-y}) - (C_1e^{y} + C_2e^{-y} - 7) = 7$