



SON TESLİM TARİHİ: Çarşamba 27 Nisan 2016 saat 12:00'e kadar.

Definition. Let $f, g : [\alpha, \beta] \rightarrow \mathbb{R}$ be piecewise continuous functions. The *inner product* of f and g is

$$\langle f, g \rangle = \int_{\alpha}^{\beta} f(x)g(x) dx.$$

Egzersiz 12 (The L^2 -Inner Product). Show that the inner product satisfies the following conditions for all piecewise continuous functions $f, g, h : [\alpha, \beta] \rightarrow \mathbb{R}$ and for all $\lambda, \mu \in \mathbb{R}$:

- (a) [5p] $\langle f, f \rangle \geq 0$;
- (b) [5p] $\langle f, g \rangle = \langle g, f \rangle$;
- (c) [5p] $\langle \lambda f + \mu g, h \rangle = \lambda \langle f, h \rangle + \mu \langle g, h \rangle$; and
- (d) [5p] $\langle f, \lambda g + \mu h \rangle = \lambda \langle f, g \rangle + \mu \langle f, h \rangle$.

Egzersiz 13 (Fourier Series). Let $f : [-1, 1] \rightarrow \mathbb{R}$ be given by

$$f(x) = \begin{cases} 0.4 & |x| \leq \frac{1}{2} \\ 1 & |x| > \frac{1}{2} \end{cases}.$$

- (a) [5p] Sketch the graph of $f(x)$, for $-1 \leq x \leq 1$.
- (b) [65p] Find the Fourier series of $f(x)$ on $[-1, 1]$.
[HINT: $L = 1$] [HINT: Your final answer should be “ $f(x) \sim \text{????????????}$ ”]
- (c) [10p] Sketch the graph of “the Fourier series of $f(x)$ on $[-1, 1]$ ”, for $-3 \leq x \leq 3$.

Ödev 6'nın çözümleri

- 11. If $\lambda < 0$, then there are no non-trivial solutions.
- If $\lambda = 0$, then there are no non-trivial solutions.
- Let $\lambda > 0$. The eigenvalues are

$$\lambda_n = \left(\frac{(n + \frac{1}{2}) \pi}{L} \right)^2$$

for $n = 0, 1, 2, 3, \dots$. The corresponding eigenfunctions are

$$X_n(x) = \sin \frac{(n + \frac{1}{2}) \pi x}{L}.$$