2015–16 **MAT372 K.T.D.D. – Ödev 8** N. Course

SON TESLİM TARİHİ: Çarşamba 4 Mayıs 2016 saat 12:00'e kadar.

Egzersiz 14 (Fourier Transforms). Let $\mathcal{F}$ denote the Fourier Transform operator (with respect to x),

$$F(\omega,t) = \mathcal{F}\big[f\big](\omega,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x,t)e^{-i\omega x}\ dx.$$

Suppose that $\lim_{x\to\pm\infty} f(x,t) = 0$

(a) [30p] Show that

$$\mathcal{F}\left[\frac{\partial f}{\partial x}\right] = i\omega\mathcal{F}[f].$$

(b) [10p] Deduce that

$$\mathcal{F}\left[\frac{\partial^2 f}{\partial x^2}\right] = -\omega^2\mathcal{F}[f].$$

(c) [60p] Let $a > 0$. Define $g : \mathbb{R} \to \mathbb{R}$ by

$$g(x) = \begin{cases} 0 & |x| > a \\ 1 & |x| < a. \end{cases}$$

Calculate the Fourier Transform of g.

Ödev 7'nin çözümleri

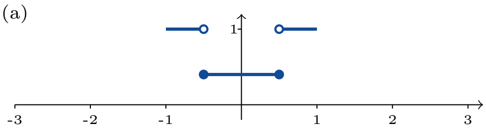
12. (a) Clearly $\langle f, f\rangle = \int_\alpha^\beta (f(x))^2\ dx \geq 0$.

(b) That $\langle f, g\rangle = \int_\alpha^\beta f(x)g(x)\ dx = \int_\alpha^\beta g(x)f(x)\ dx = \langle g, f\rangle$ is trivial.

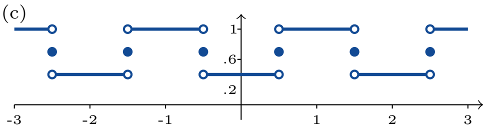
(c) $\langle \lambda f + \mu g, h\rangle = \int_\alpha^\beta (\lambda f(x) + \mu g(x))h(x)\ dx = \lambda \int_\alpha^\beta f(x)h(x)\ dx + \mu \int_\alpha^\beta g(x)h(x)\ dx = \lambda\langle f, h\rangle + \mu\langle g, h\rangle$.

(d) Follows immediately from (ii) and (iii).

13. (a)



(c)



(b) $f(x) \sim \frac{7}{10} - \sum_{k=1}^{\infty} \frac{6}{5k\pi} \sin\left(\frac{k\pi}{2}\right)\cos(k\pi x)$